Valuation of Near-Market Endogenous Assets*

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Abstract

For many kinds of assets, the growth rate of the real asset stock is a nonlinear function of the economic owner's decision whether to invest or extract the asset. Examples within the economy are primarily biological assets, both privately owned (such as those found in aquaculture and agriculture) and publicly owned or regulated (such as fish stocks, and in some case, timber stocks.) Optimal exploitation of the asset necessitates that the future possible growth rates in the asset must be considered when determining the optimal amount of extraction today. In this sense, the level of the asset is determined by the economic owner or regulator and is thus said to be endogenous. This paper considers existing methods for the valuation of these endogenous assets when observed transaction prices are lacking. In particular, we consider valuation in a near-market context, whereby the the economist can only observe income flows from the asset. This near-market approach to asset valuation is particularly important for environmental accounting when transaction prices for the asset or the right to exploit the asset are lacking. We give sufficient restrictions on the revenue and cost structure of the firm in order to permit asset valuation based on average profits. In an emprical application, we combine economic and biomass data to value the US Bering Sea crab fisheries.

*The views expressed herein are those of the authors and not those of the Bureau of Economic Analysis or the Department of Commerce.

1 Introduction.

Obtaining an accurate valuation of a natural resource is central to the United Nations System of Environmental Economic Accounts (SEEA). A revised SEEA central framework is due for publication in 2012. The valuation of endogenous natural assets is an important class of natural resource assets. By endogenous assets we mean assets whose stocks in part vary by an internal growth process. One such natural asset that has received a lot of attention in the economics literature is fisheries. The treatment of fisheries is cast in a capital theoretic context in which the optimal stock is determined by discounting the benefits and costs over time subject to the constraints of the growth process. See for example Scott (1954), Gordon, (1954), Crutchfield and Zellner (1962), and Turvey (1964); the role of these treatments is described in Munro (1992). The dynamic optimization problem is an offshoot of the Hotelling (1931) and the models described in Clark (1985).

Other types of natural assets to which this framework could apply is forests, wild animal preserves, and to some extent this is the problem underlying cultivated assets such as herds, vineyards and so on.

The effort in the literature is directed toward determining optimal stock and optimal extraction. In this paper the focus is on the determination of the asset value given the optimal stock and extraction. From this asset value we can then obtain a real measure of the stock of capital. The standard approach to measuring the stock of capital is to take the initial value, add investments and subtract depreciation, obsolescence and so on. For the endogenous assets considered herein may of these additions and subtractions are inherent - the growth process for fish considers the birth and death of the stock as well as the extraction that is part of the production process.

Hotelling valuation (HV) is conventionally applied to assets for which the growth in the quantity of the asset is independent of the decision to extract or invest in the asset.[1] For example, Miller and Upton (1985) apply HV to oil and gas reserves, while the Integrated Economic-Environmental Satellite Accounts

[1] We refer the reader to the Appendix for a brief overview of Hotelling valuation applied to these conventional assets when marginal profits are constant.

(IEESAs, 1994a,1994b) produced by the BEA apply HV to both subsoil hydrocarbon and mineral assets in the US economy. Here the physical quantity of asset is independent of how much the firm chooses to extract; indeed, the physical quantity of the asset below the ground is fixed. Yet independence is clearly not appropriate for other assets in the economy. For example, the growth in many cultivated biological assets, such as fish raised in fish farms, is dependent on whether or not the firm chooses to extract some of the asset. We describe these assets as *endogenous* in the sense that the firm simultaneously chooses a sequences of extraction levels and a long-run asset level in order to maximize the value of the asset. In this paper we consider Hotelling valuation (HV) for assets with this type of endogenous growth.

Examples of endogenous assets within the economy are primarily biological assets, both privately owned (such as those found in aquaculture and agriculture) and publicly owned or regulated (such as fish stocks, and in some cases, timber stocks.) Typically, growth is specified as a function of the level of the asset stock, so that the economic owner or regulator will simultaneously choose an extraction level and a asset quantity to maximize the present value of future profits.[2] Value added generated by these assets that fall within this category contributed just over 1.02% of the US economy over the 1999-2008 period, as the table below demonstrates. For comparison, oil, gas and mining extraction together made up 1.13% of the economy. The value added generated by the entire Agriculture Forestry and Fishing industry was $136,413 million for 2009 (current dollars). If we were to apply a back-of-the-envelope present value calculation, assuming that all future revenue from the sector would remain the same (in 2009 dollars), this would place the present value between 3.4 and 1.9 trillion dollars, using a discount rate of 4% and 7%, respectively. For comparison, the total market value of all corporations traded on the NYSE was 12 trillion in August 2010.

[2] In some cases growth is specified as a function of the age of the stock. For example Clarke and Reed (1989) specify the natural growth in trees as a function of the age of the tree.

Value Added of selected Industries as % of GDP

Year	Agriculture, forestry, fishing, and hunting	Farms	Forestry, fishing, and related activities	Mining	Oil and gas extraction	Mining, except oil and gas
1998	1.14	0.90	0.24	0.92	0.45	0.32
1999	0.99	0.76	0.23	0.88	0.46	0.31
2000	0.96	0.74	0.22	1.09	0.68	0.28
2001	0.96	0.74	0.22	1.16	0.71	0.27
2002	0.89	0.68	0.21	1.03	0.63	0.26
2003	1.04	0.83	0.21	1.21	0.80	0.25
2004	1.20	1.00	0.21	1.34	0.90	0.26
2005	1.01	0.81	0.20	1.52	1.02	0.29
2006	0.91	0.69	0.22	1.71	1.10	0.30
2007	1.04	0.83	0.22	1.72	1.07	0.31
2008	1.13	0.91	0.22	2.13	1.41	0.34
mean	1.02	0.81	0.22	1.34	0.84	0.29

HV for endogenous assets combines economic theory, describing optimal firm behavior given a set of constraints, with biological models of the natural world. In particular, we will require mathematical functions that describe how the asset grows in response to the extractive decisions of the firm. The models for solving the firm's problem are well-established in the literature (see Clark, 2010, for a detailed overview of so-called "bioeconomic" methods), and this paper will draw on many of the theories already well established in the literature. In this sense we are not re-inventing the wheel. Rather, the purpose of the paper is outline these methods, and discuss what assumptions on the models would be necessary for us to derive accurate valuation of these assets *given an observed income stream* from the asset. The issue is thus seeing how far firm- or industry-level accounting information can be used to obtain values for endogenous assets. Such assets will be called "near-market" assets; although transactions prices for the asset itself are lacking, we can observe the price and quantity of the goods flowing from the asset. We will begin by stating the firm's extraction problem within a general and rich model that permits various cost and market structures on the economic side, and we will initially remain agnostic about the biological growth function. Having solved the model, we will then outline the weakest possible assumptions that facilitate asset valuation given the observed income stream from the asset.

Typically data for the marginal profit of extraction across a range of different extraction rates is lacking. This would inform us about the profitability of the firm as we change the amount extracted, and in a dynamic setting, knowledge of the marginal profitability of extraction is crucial as the firm will select a sequence of current and future extraction rates in order to maximize the value of the asset. Instead, one can only usually observe average profit per unit from extraction of the asset through conventional cost and revenue per unit reports. In order to infer valuation based on this data, one must impose simplifying assumptions between the observed average return and the unobserved marginal return in order to derive a valuation from the data. (See for example, Miller and Upton, 1985, and Nordhaus and Kokkelenberg, 1999). In what follows we will outline sufficient assumptions for valuation from average per-unit profit.

Another important factor in solving the optimal control problem of the firm is the ownership structure of the asset. The firm may take into account how its extraction this season will affect the future stock level, which in turn will affect the firm's profits in beyond the current period. When the number of owners is sufficiently small, a profit-maximizing firm will take the affect of its current extraction on the future stock level into account. Consider an aquaculture firm, for example. The firm has exclusive rights to extract fish from its private tanks. In contrast, when ownership is sufficiently sparse, so that the effect of the firm's extraction on the asset level is very small, a profit-maximizing firm will not take the affect of its current extraction into account. For example if a wild resource is permitted to be extracted by some firms through regulation, then the effect of the firm's extraction on the stock level is arbitrarily small. If the firm's take is sufficiently small the firm may not take into account its current extraction on the future stock of the asset. Predictably this leads to the firm adopting a higher level of extraction in the short run. Yet increasingly extraction of these assets is regulated by a central authority. For these assets, the total extraction (across all firms permitted access) is set by the authority is order to attain a long-run asset stock level. Of course, the target long-run level may not be the profit-maximizing level of the asset. Thus we will argue that the first best option in this eventuality is to operationalize the regulator's stated goal - if she has one - into an objective function. A second best option will be to assume that the authority acts as the optimizer buy choosing the optimal stock level to maximize firm profits. In either case, we may use firm-level accounting data to value the resource.

The rest of the paper is organized as follows. In section 2 we outline the optimal control theory to be used to value the near-market assets. In this section we will also outline sufficient conditions on firm cost and revenue for valuation to be possible based on average profitability data. In section 3 we will consider the specific example of the Bering Sea crab fisheries. Section four concludes.

2 Theory.

We begin by stating the dynamic optimization problem of the representative firm. Much of what we summarize here has been gleaned from Clarke (2010). Before we begin it is instructive to define the following notation.

Notation. We define the following.

$$
\begin{aligned}
H_t &: \quad \text{extraction (or ``harvest'') at time } t. \\
X_t &: \quad \text{asset stock at time } t. \\
V_t &: \quad \text{value of asset stock at time } t. \\
\delta &: \quad \text{discount rate.} \\
g\left(X_t\right) &: \quad \text{growth function for asset stock.} \\
p\left(H_t\right) &: \quad \text{inverse demand function for } H_t. \\
c\left(H_t, X_t\right) &: \quad \text{firm cost function.}
\end{aligned}
$$

Firm cost $c\left(H_t, X_t\right)$ is a function of both the harvest H_t and the asset stock X_t at time t. As a function of H_t we are permitting standard increasing marginal costs. Costs are also permitted to be a function of the asset size X_t, reflecting "search costs": As abundance of many biological stocks increases, the costs associated with searching for the asset decrease. This aspect may be more relevant for "wild" stocks such as fish as opposed to cultivated biological assets. For search costs we would expect $\partial c(H_t, X_t)/\partial X_t < 0$. Similar flexibility is permitted in models of optimal extraction of non-renewable assets. For example, when an oil well is new, oil extraction is cheap as the oil is under pressure and can be extracted easily (see, for example, chapter 3 of Nordhaus and Kokkelenberg, 1999; Miller and Upton, 1985). In this case $\partial c(H_t, X_t)/\partial X_t < 0$.

We permit the price received by the firm $p(H_t)$ to be a function of the quantity harvested, thereby permitting the firm to exhibit market power. Firm profits are given by

$$\pi(H_t, X_t) = (p(H_t) - c(H_t, X_t)) H_t$$

Note that profit is not necessarily linear in extraction H_t. Valuation of the asset boils down to

$$V_0 = \int_0^\infty e^{-\delta t} \pi(H_t, X_t)\, d_t = \int_0^\infty e^{-\delta t} \left[(p(H_t) - c(H_t, X_t)) H_t \right] d_t. \tag{1}$$

We must solve for $\{H_t\}_{t=1}^\infty$ and $\{X_t\}_{t=1}^\infty$. The optimal control problem becomes a variation in Hotelling (1931).

2.1 Generalized Hotelling Model.

The firm's problem is to maximize the objective function;

$$V_0 = \max_{\{H_t\}_{t=1}^\infty} \int_0^\infty e^{-\delta t} \pi(H_t, X_t)\, d_t, \tag{2}$$

subject to the constraints

$$\begin{aligned} \dot{X}_t &= g(X_t) - H_t, \\ X_t &\geq 0, \\ 0 &\leq H_t \leq H_{\max}. \end{aligned}$$

The current value Hamiltonian is

$$\mathbf{L}(H_t, X_t, \psi) = \pi(H_t, X_t) + \psi_t (g(X_t) - H_t) \tag{3}$$

where $\psi_t := e^{\delta t}\lambda_t$, and λ_t is the shadow price for the present value Hamiltonian. Then the first order conditions (FOCS) are

$$\begin{aligned}\frac{\partial \mathbf{L}(H_t, X_t, \psi)}{\partial H_t} &= 0\\ \frac{\partial \mathbf{L}(H_t, X_t, \psi)}{\partial X_t} &= \delta\psi_t - \dot{\psi}_t\end{aligned}$$

which under (3) imply that

$$\frac{\partial \pi(H_t, X_t)}{\partial H_t} = \psi_t, \tag{4}$$

$$\dot{\psi}_t = \delta\psi_t - \frac{\partial \pi(H_t, X_t)}{\partial X_t} - \psi_t g'(X_t), \tag{5}$$

respectively. Eq.(4) says that the marginal value of harvesting must equal the marginal product of the asset in equilibrium. Eq.(5) can be re-written as

$$\delta = \frac{\dot{\psi}_t}{\psi_t} + \frac{\partial \pi(H_t, X_t)}{\partial X_t \psi_t} + g'(X_t)$$

which says that the marginal return to delaying extraction (capital gains on the asset $\frac{\dot{\psi}_t}{\psi_t}$ plus the effect on profit $\frac{\partial \pi(H_t, X_t)}{\partial X_t \psi_t}$ plus marginal changes in volume $g'(X_t)$) must be equal to the rate of time preference. Combining the results we have

$$\dot{\psi}_t = (\delta - g'(X_t))\frac{\partial \pi(H_t, X_t)}{\partial H_t} - \frac{\partial \pi(H_t, X_t)}{\partial X_t}$$

Note that if $\frac{\partial \pi(H_t, X_t)}{\partial X_t} = 0$, the above implies that the shadow price changes at rate $\delta - g'(X_t)$.

Equilibrium. Equilibrium is defined by setting $\dot{\psi}_t = 0$ and $\dot{X}_t = 0$, which give us our solutions X_t^* and H_t^*:

$$g'(X_t^*) + \frac{\partial \pi(H_t^*, X_t^*)/\partial X_t^*}{\partial \pi(H_t^*, X_t^*)/\partial H_t^*} = \delta \tag{6}$$

$$g(X_t^*) = H_t^* \tag{7}$$

Together these two equations characterize the amount extracted H_t^* as well as the asset stock X_t^* in equilibrium. (H_t^* is often referred to as the "economic sustainable yield".) Exact expressions require specifying more a precise profit function $\pi(H_t, X_t)$ and growth function $g(X_t)$. We address each issue in the following two sub-sections. After this, we will discuss transition to the equilibrium. We will argue that the transition to equilibrium depends on the market structure.

2.1.1 Market and cost structure.

The simplest case to consider is that of a price-taking firm with constant marginal costs. Thus the marginal profit of each unit is the same, and hence - in the absence of expected changes in price and cost per unit - the net present value of the asset is a linear function of current total profits. (See, for example, Miller and Upton, 1985.) Indeed, the assumption is often exploited in order to produce valuations of the near-market assets based on total profit figures (see, for example, BEA's Integrated Economic-Environmental Satellite Accounts, 1994a, 1994b, and Nordhaus and Kokkelenberg, 1999).

Condition 1 *(Price Taking Firm):* $\partial p(H_t)/\partial H_t = 0$

Condition 2 *(Constant Marginal Cost):* $\partial c(H_t, X_t)/\partial H_t = 0$, *i.e. marginal cost for an additional unit extracted is independent of* H_t.

In equilibrium, $\dot{\psi}_t = 0$, so that for a constant marginal cost price taker we have $\partial \pi(H_t, X_t)/\partial H_t = p_t - c(X_t)$, and hence in this case (6) becomes

$$g'(X_t^*) + \frac{c'(X_t^*)\, g(X_t^*)}{p - c(X_t^*)} = \delta$$

In addition, the cost per unit is permitted to be a function of the asset stock X_t. This reflects "search costs": For firms in which extraction requires a search - such as fisheries - it is natural to expect that costs are some decreasing function of the abundance of the asset. For example, a firm with constant marginal costs (in H_t) may have a cost function specified as $c(X_t) = X_t^{-1}c$ (see for example, chapter 1 in Clark, 2010). Yet from a valuation point of view, this assumption is troublesome since it would imply knowledge of the c parameters in order to produce valuation of the stock when X_t is transitioning to equilibrium. Hence, we shall also assume that

Condition 3 *(No search costs):* $\partial c(H_t, X_t)/\partial X_t = 0$

Moreover, there is empirical evidence to doubt that search costs are binding for "wild" assets such as fisheries (Fretwell and Lucas, 1970; see also section 3.3 of Clark, 2010).

Thus under this additional assumption (6) becomes

$$g'\left(X_t^*\right) = \delta, \tag{8}$$

which together with $g\left(X_t^*\right) = H_t^*$, characterizes the stock level and extraction for a given growth function $g\left(X_t\right)$. Thus if the asset is at equilibrium X_t^*, valuation of the asset simplifies to

$$V_0 = \int_0^\infty e^{-\delta t}\pi\left(H_t^*\right) d_t = \left(p_0 - c_0\right) g\left(X_t^*\right) \int_0^\infty e^{-\delta t} d_t, \tag{9}$$

where the second equality holds if we assume that future price and cost per unit extracted is the same as current price and cost per unit extracted.

2.1.2 Variation in Productive Efficiency.

Much of the empirical literature has examined the cause of variation in efficiency both across time and firms. For example, Kompas and Che (2005) showed how the introduction of a transferable quota system resulted in aggregate cost reductions in the Australian South East Trawl fishery. Kirkley, Squires and Strand (1998) and Alvarex and Schmidt (2006) examine and test various hypotheses for the variation in technical efficiency between boats.

Changes in technical efficiency can be incoporated into the model through the per-unit costs function $c\left(H_t, X_t\right)$. Notably the per-unit cost function $c\left(X_t, H_t\right)$ in the general model outlined above is time-invariant. However care must be taken to distinguish between movements along the cost function from shifts in the cost function depending on the soruce of increases in efficiency. For example, reductions in marginal cost due to a policy-maker restricting catch or to increased stock levels would represent a movement along the cost function. Changes in technology and improvements in managerial skill would represent a shift in the cost function. This could be accounted for by interacting an additional variable with the cost function, such as $\alpha\left(t\right) \cdot c\left(H_t, X_t\right)$. When using solving the optimal control problem to value a stock, any foreseable secular trends in the cost function would have to be incorporated into the firm's problem, complicating the optimal control problem by adding an additional state variable $\alpha\left(t\right)$.

2.1.3 Uncertainty.

The model outlined above is deterministic. In practice the firm faces a variety of risks; prices can vary, costs can vary, the growth function may be unknown, and there may be unforeseeable factors that affect the stock (e.g., "catastrophic losses" in fisheries). The uncertainty associated with these variables would likely affect the price a buyer would be willing to pay for the asset.

The textbook approach to incorporating uncertainty into the NPV formula is to adjust the discount rate upwards to reflect risk. The income streams going forward used in the NPV formula are then the expectation of the income streams (given that in the stochastic world we no longer know the income streams with certainty). Clarke and Reed (1989) show formally that bioeconomic asset valuation in a Gaussian framework is equivalent to valuation using an adjusted discount rate and the expected income streams. This naturally raises the issue of what discount rate to use in practice. Further complicating the issue is the emerging consensus that the discount rate itself varies over time as investor appetite for risk varies over the business cycle (Cochrane, 2010). This would suggest that the discount rate needs updating over the business cycle.

The selection of the appropriate discount rate for the costs associated with climate change at the macro level has been the subject of intense debate. Nordhaus (1997) characterizes two contrasting approaches, the 'descriptive approach' "asks what combination of parameters can rationalize existing rates of returns" on the assets we observe in the market place. The 'prescriptive approach' "begins with a view about time preference and inequality aversion and from this concludes what the appropriate discount rate is." The latter approach may be characterized as normative, the former as positive approaches to select the discount rate. The prescriptive discount rate is often selected to be low on the basis of ethical arguments, as in the Stern Review (Stern, 2006). As pointed out by Nordhaus (1997), the two views can be reconciled by considering the scarcity of the asset in question in the future. That said, Nordhaus (2007) criticizes the Stern review discount rate as being excessively low.

As we show below in our application, selection of the discount rate can have a massive impact on the value of the asset in the Hotelling model. Practitioners thereby need to exercise care when selecting the appropriate discount factor.

We next consider candidate growth functions $g(X_t)$ for the asset.

2.1.4 Growth Functions.

Valuation hinges on the growth function $g(X_t)$. (The growth function across specific range of X_t is sometimes referred to as the "sustainable yield curve; e.g. paragraph 5.83 in Chapter 5 of SEEA and Figure 5.4.1. The extraction level is sustainable because change in the stock level can be harvested each period without affecting the stock level.) In a data rich environment it may be possible to estimate such a function by nonparametric techniques. In the absence of such data, we may have to rely on a parametric specification that permits limited flexibility. A commonly used growth function in biological applications is the logistic function

$$g_{\mathrm{L}}(X_t) := rX_t(1 - X_t/K),$$

where K is the maximum asset size. Below we graph the function $g(X_t)$ against X_t for various r and K values.

Unharvested assets approach the steady state level or "carrying capacity" K. The parameter r controls the height of the function, and therefore it governs the speed at which the asset approaches the carrying capacity K in the absence of harvesting. For stocks that are in transition (see section 2.2 below), r governs the rate at which the stock approaches the equilibrium level X_t^* when $X_t < X_t^*$. It also governs the amount of the resource that can be extracted sustainably. The greater r is (all else being equal), the greater the growth rate in the stock for each X_t, and hence the greater the amount that can be sustainably harvested in each period.

Under this parametric structure we have two parameters to estimate: r and K. Thus we require at east two biological data points from which to fit the curve. It is common for asset owners to collect data on the "maximum sustainable yield" (MSY) which in terms of the given growth function is

$$H_{\mathrm{MSY}} = g(X_{\mathrm{MSY}}), \text{ where } X_{\mathrm{MSY}} \text{ maximizes } g(X_t)$$

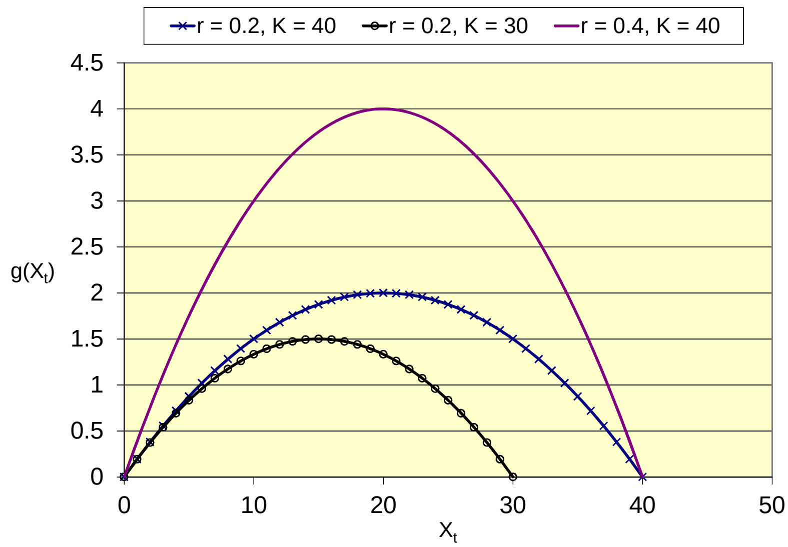

Figure 1: Logistic Functions: $g_{\mathrm{L}}(X_t) := rX_t(1 - X_t/K)$

In terms of the figure above, H_{MSY} denotes the maximum height of the function. It is also common for the asset owner to know the stock level corresponding to the MSY, i.e. X_{MSY}. These two data points would be sufficient to identify the logistic growth curve.

As the number of data points is increased, more complicated growth specifications can be considered. For many biological resources, severe harvesting can lead to a population collapse (Hutchings, 2000), a result prohibited by the logistic function. (For any non zero stock level less than the carrying capacity, growth is positive in the logistic function.) A commonly cited example is fisheries. It is thus common for growth functions to permit critical depensation, meaning there is a minimum non-zero stock size, below which natural growth is negative. Thus if the population falls below this "minimum viable population" (MVP), the stock will collapse. The growth function displayed below features a minimum viable population. It is constructed by simply adding a constant to the logistic function. Here the MVP is 4 units. For any $X_t \leq 4$ we have $g_{\mathrm{L}}(X_t) \leq 0$.

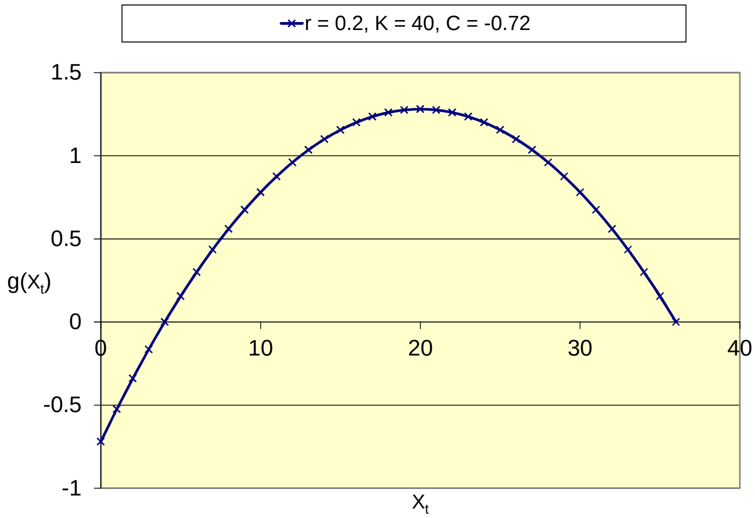

Figure 2: Logistic Function with Depensation: $g_{\mathrm{L}}\left(X_t\right) := rX_t\left(1 - X_t/K\right) + C$

Next we consider a skewed logistic function of the form

$$g_{\mathrm{L}}\left(X_t\right) := rX_t^{\alpha}\left(1 - X_t/K\right)$$

For $\alpha > 1$ the function is skewed to the right, whilst for $\alpha < 1$ it is skewed to the left.

We have considered a couple of prominent growth functions used in the bionomic literature. However, we can accommodate a wide variety of functional forms. The only sufficient property that is necessary is differentiability of the growth function. Hence the growth function must be smooth. It is also useful to assume a single global maximum to ensure there is a single stock level that corresponds to the maximum sustainable yield.

2.1.5 Graphical representation of the long-run equilibrium

Below we graph the bioeconomic equilibrium for the price taking firm with constant marginal costs and no search costs. The equilibrium condition is given in (8) above. In this case, the stock level is selected so that the slope of the growth function is equal to the discount rate. In the figure below we set the

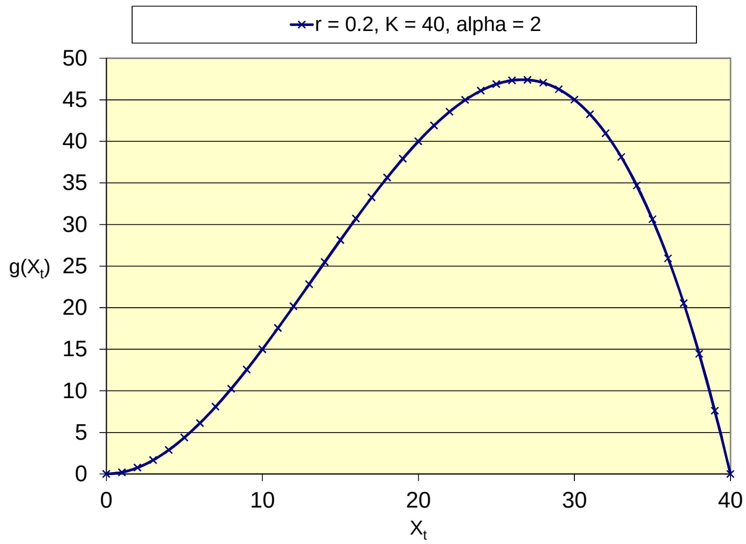

Figure 3: Skewed Logistic Function: $g_{\mathrm{L}}(X_t) := rX_t^{\alpha}(1 - X_t/K)$

discount rate to 5.5%. Note that in this example, the sustainable resource rent is a scaling of the growth function because price and cost is constant.

Note that the optimal stock level in this case ($X_t^* = 15$) is below the stock the level associated with the maximum sustainable yield ($X_{\mathrm{MSY}} = 20$). This will be true for any positive discount rate, and it reflects the opportunity cost of asset ownership. In equilibrium, the rate if return on the asset must equate to that the owner could receive by selling the asset and investing the proceeds. In the example above, the rate of return on the unharvested asset is 5.5% due to endogenous growth (recall that price is constant here) when $X_t = 15$.

In this simple example the optimal stock and harvest is independent of the price per unit and cost per unit (provided the marginal revenue per unit is positive). Both variables are assumed constant in this simple case, so that the firm does not consider the effect of its behavior on either variable. We will now consider relaxing this assumption.

We consider the more complicated model whose equilibrium is given in (7). That is, let us introduce search costs to the model. In this case the firms costs

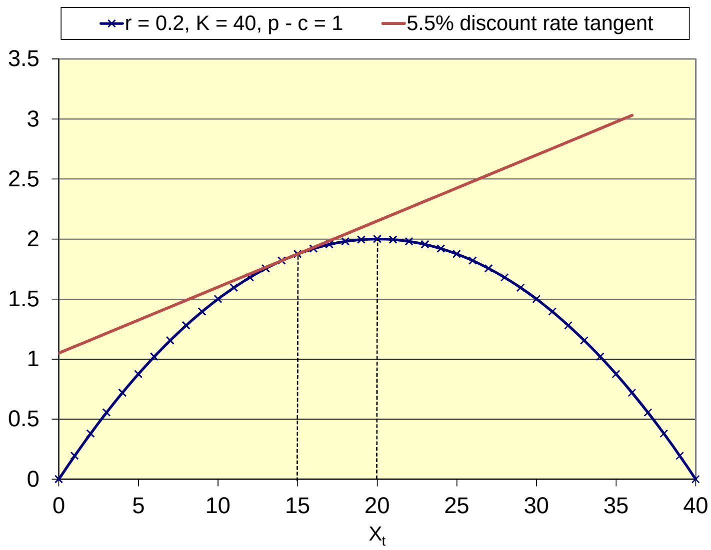

Figure 4: Sustainable Resource Rent and Tangency Condition. The sustainable resource rent is $(p-c)\,g\,(X_t)$, $\;g\,(X_t) = rX_t\,(1 - X_t/K)$

are affected by the stock level. More specifically, the firms costs are decreasing in the stock level, reflecting the fact that it is easier to harvest the stock when it is more plentiful. (A reasonable assumption for many wild resources.) Our equilibrium in this case is

$$g'\left(X_t^*\right) + \frac{c'\left(X_t^*\right) g\left(X_t^*\right)}{p - c\left(X_t^*\right)} = \delta$$

Recall that the cost function $c\,(X_t)$ reflects the dependence of costs on stock size.

In addition, the cost per unit is permitted to be a function of the asset stock X_t. This reflects "search costs": For firms in which extraction requires a search - such as fisheries - it is natural to expect that costs are some decreasing function of the abundance of the asset. For example, a firm with constant marginal costs (in H_t) may have a cost function specified as $c\,(X_t) = X_t^{-1}c$, and hence $c^{'}\,(X_t) = -X_t^{-2}c$. To see the graphical solution, note that the firm's sustainable

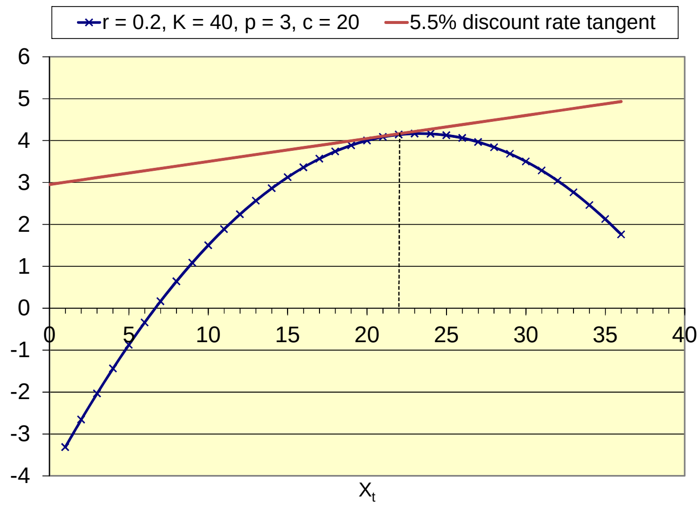

Figure 5: Sustainable Resource Rent and Tangency Condition. The sustainable resource rent is $(p-c)\,g(X_t)$, $g(X_t) = rX_t(1-X_t/K)$, $c(X_t) = cX_t^{-1}$

resource rent for a given stock level X_t is

$$(p - c(X_t))\,g(X_t) = \left(p - \frac{c}{X_t}\right) rX_t \left(1 - \frac{X_t}{K}\right)$$

so that the solution is simply given by the point where this function has slope equal to δ. See figure 5.

The introduction of search costs pushes the optimal stock level to the right, simply because due to the reduction in cost by doing so. If the search costs are sufficiently great, and the discount rate sufficiently low, the optimal stock level can be to the right of the maximum sustainable yield, as occurs in this example. (Note that the MSY for the growth function in the example in the figure is 20.) This is because the effect of search costs outweighs the effect of discounting in this particular example.

2.2 Transition to equilibrium.

Although we have solved for the asset level and extraction rate in equilibrium for our general model, we have not yet discussed how the transition to equilibrium occurs.

When the profit function is linear in H_t, the firm simply stops extraction or extracts the required amount to get to X_t^* as fast as possible. This would be the case for price-taking firms with constant marginal cost, or in general firms with profit functions linear in H_t. The extraction rate H_t is said to be a *switching function*, where

$$H_t = \begin{cases} 0 \text{ if } X_t < X_t^* \\ H_{\max} \text{ if } X_t > X_t^* \end{cases}.$$

H_t obeying the above insures that the equilibrium stock level is reached as fast as possible (c.f. eq. (2.46) in Clark, 2010).

However, if the owner has market power, then as the supply is restricted to conserve and grow the asset base, the price is driven up. At some point, the price reaches such a level that it becomes optimal for the owner to extract. Thus under market power - and in more general cases where the profit function is non-linear in H_t - there is a smooth transition to the optimal stock level. This smooth transition can be solved for recursively, by solving backwards from the equilibrium conditions. In general this recursion is dependent on key parameters governing cost and price that will be unobservable to the economist. However, because the assumption of market power is untenable for many of the firms we are considering, we will not pursue this solution any further. We refer the reader to chapter 4 of Clark (1985) for a more detailed explanation of how the optimal transition path can be derived in a more complicated model with market power. Smooth transition may also occur if there are search costs. As the asset stock is conserved, the marginal cost of extraction decreases, and again at some point it becomes optimal to extract. However, as stated above, due to both the lack of empirical and theoretical relevance of search costs and the complications it would create for valuation, we shall continue to maintain zero search costs.

2.3 Separation of Asset Owner and Extracting Firm

The dynamic optimization problem has thus far only considered the case in which the extracting firm has exclusive property rights to harvest the asset stock. The only influence on the stock level is the firm's own extraction, and hence the firm only takes into account its own extraction when solving the dynamic optimization problem. This precludes cases in which other factors that are not under the control of the firm can affect the stock level. In particular it precludes the case in which other firms are harvesting the stock.

In practice for many wild biological resources no single firm has exclusive rights to harvest. In this case, the effect of any single given firm's extraction rate on the growth of the asset becomes diluted as the number of firms accessing the asset increases. For a large enough number of firms, each firm will disregard its own extraction rate when optimizing future profits,and each firm will also disregard the extraction rate of other firms. (In practice this situation closely corresponds to unregulated wild resources, whereby any firm can enter and harvest the stock.) In this case the dynamic aspect of the optimization problem disappears, and the firm simply selects the harvest rate at which marginal cost equals marginal revenue. The value of the stock can then be derived under this so-called "open access" situation given the growth function of the stock. Typically the asset value is far below that in the case where a single firm has exclusive property rights. In many cases the wild resource is harvested until it disappears. However, open access is becoming increasingly scarce as governments act to preserve wild resources. Typically this is achieved by the regulator (henceforth "owner") limiting the amount harvested over a given period.

In this subsection we consider the case in which the owner of the stock distributes rights to harvest to extracting firms in each period. The owner manages the sequence of harvests according to some objective function, and under the assumption that the harvesting rights are exhausted once distributed (that is, the extractor does not harvest any less than he is permitted). We will consider the case of a continuum of competing firms, so that the individual firm does not consider the impact of its harvest on the growth rate of the asset, nor does it interact strategically with other firms when choosing its own extraction rate. Instead, we assume that a regulator sets total H_t according to her objective function, before allocating out individual extraction rights $h_{i,t}$ to various firms $i = 1, ..., n$, subject to $H_t = \sum_{i=1}^{n} h_{i,t}$. Obviously we must make an assumption about the regulator's objective function.

When the resource manager has a stated extraction policy, the best option may be to use the stated policy to determine the asset value regardless of whether the stated policy is economically optimal. In the example given in the following section, the resource manager explicitly states its control policy for H_t as a function of the underlying stock size X_t, so that we can directly predict how H_t will change over time (given of course some growth function.) Often these so-called "reference point" strategies are set without regard to optimizing the value of the asset, but are rather ad-hoc rules for managing the stock to a stock level somewhere in the vicinity of X_{MSY} (Hilborn, 2002). Typically these reference point rules are linear functions of the underlying stock level. For example, a linear reference point rule is as follows.

$$H_t = \begin{cases} 0 \text{ if } X_t \leq a \\ (b-a)^{-1}(H_{\max} - a) \text{ if } a < X_t \leq b \\ H_{\max} \text{ if } X_t > b \end{cases}$$

Note that under this reference point rule, if $b = X_{\text{MSY}}$, then for all $X_t > X_{\text{MSY}}$ there is unrestricted extraction of the asset. In this way convergence to the steady state may not be smooth, let alone be achieved at all. Another reference point rule that ensures convergence to X_{MSY} is as follows.

$$H_t = \begin{cases} 0 \text{ if } X_t \leq a \\ (b-a)^{-1}(H_{\text{MSY}} - a) \text{ if } a < X_t \leq b \\ H_{\text{MSY}} \text{ if } X_t > b \end{cases} \tag{10}$$

Although this rule does not maximize the asset value, it does ensure smooth convergence to X_{MSY}.

In the absence of a stated policy, the economist must make an assumption about the regulator's objective function. In such situations we may wish to assume that the regulator's goal is to maximize firm profits, and has knowledge of the costs and market structure for the good in question. Such an assumption may rightly be perceived as heroic. Yet we require *an* assumption about the objective function of the agent controlling the asset, and in the absence of a better assumption, we will assume that the regulator has profit maximization as her primary goal. In this case, she chooses H_t^* according to (6) and (7).

3 Application.

In this section we use biological and accounting data provided by the National Oceanic and Atmospheric Administration (NOAA) in order to value an Alaskan crab fishery. NOAA provides estimates of stock levels, catch levels, H_{MSY} and X_{MSY} as well as reference points rules (10) for determination of the fishing quotas ("total allowable catch"). NOAA also provides limited accounting data on the costs and revenues realized by firms harvesting the crabs.

NOAA conducts surveys of many wild fish stock populations, but economic data associated with these stocks is lacking. However, recipients of fishing quotas in the Alaskan King Crab fishery are required to fill out economic surveys administered by the NOAA. These surveys cover the revenues from Alaskan crab harvesting, and a substantial proportion of the costs. On the cost side, gasoline and capital costs are omitted from the survey. The 2010 Stock Assessment and Fishery Evaluation (SAFE) report contains this limited accounting survey data for the Alaskan Crab fisheries. The SAFE surveys are issued on an annual basis, but SAFEs previous to 2010 typically only contain data on harvests and stock levels. In what follows, we will combine the accounting data from the 2010 SAFE with biomass data from the 2009 SAFE into a working example for valuation of near-market assets under endogenous growth.

All data are at an annual frequency. Typically, the fleet harvests several species of crab across a given year. The most harvested are the Bristol Bay Red King crab, Opilio (Snow) crab, and the Tanner crab. We will value these three stocks jointly, because some of the costs from the 2010 SAFE report are not allocated to the harvesting of each species.

3.1 Growth function.

NOAA publishes detailed data on the stock levels for the Alaskan King crab, Tanner crab and Opilio ("Snow") crab stocks together with estimates of the H_{MSY} and X_{MSY}. From this we can identify a two-parameter growth function, such as the logistic function. NOAA also publishes past data on Total Allowable Catch (TAC) and realized catches for the fisheries. They also publish their stated reference point rules for determining future TACs. Given a growth function, from this data we can infer the future growth in the asset stock and TAC under NOAA's stated reference point rules for extraction of the stock to bring the stock level to X_{MSY}. Tables 1 to 3 below give the past and predicted managed changes in three crab stocks located in the Bering Sea.

Table 1: Projected Harvests and Stocks (millions lbs)
Bristol Bay Red King Crab

	X_{MSY}	H_{MSY}	X_t	H_t	$g_{\text{L}}(X_t)$
2005			**80**	**18**	
2006			**82**	**16**	
2007			**86**	**20**	
2008	75.1	24.2	**95.6**	**20.4**	*22.4*
2009	75.1	24.2	*97.6*	*24.2*	*22.0*
2010	75.1	24.2	*95.5*	*24.2*	*22.4*
2011	75.1	24.2	*93.7*	*24.2*	*22.7*
2012	75.1	24.2	*92.2*	*24.2*	*22.9*
2013	75.1	24.2	*90.9*	*24.2*	*23.1*
2014	75.1	24.2	*89.9*	*24.2*	*23.3*
2015	75.1	24.2	*88.9*	*24.2*	*23.4*
2020	75.1	24.2	*85.6*	*24.2*	*23.7*
2030	75.1	24.2	*82.3*	*24.2*	*24.0*

Entries in bold denote historical data obtained from the 2009 SAFE. Here H_t denotes the annual harvest and X_t denotes the stock level at the start of the season. Entries in standard font denote estimates for the maximum sustainable yield H_{MSY}

and the MSY stock level X_{MSY} and are obtained from the 2009 SAFE. Numbers in italics represent our projections using harvest rates H_t prescribed by the stated reference point rules from the 2009 SAFE and growth in stock levels using the logistic growth function $g_{\text{L}}(X_t)$. The reference point rule used by NOAA for the King Crab is $H_t = H_{\text{MSY}}$ because $X_t > X_{\text{MSY}}$. The stated years refer to the year in which the fishing season begins; e.g. "2008" refers to the 2008-2009 fishing season.

Note that projected extraction rates are set equal to the H_{MSY} for the King crab stock. This is because stock levels in 2008 exceed the maximum sustainable yield stock level. Thus going forward it is assumed that the TAC in each season will be set equal to the estimated $H_{\text{MSY}} = 24.2$. (This is in accordance with the stated reference point rules of NOAA in the 2009 SAFE.) We can see that as time passes the stock level is forecasted to approach the $X_{\text{MSY}} = 75.1$ rather slowly.

Table 2: Projected Harvests and Stocks (millions lbs)
Bering Strait Tanner Crab

	X_{MSY}	H_{MSY}	X_t	H_t	$g_{\text{L}}(X_t)$
2005			**86.2**	**4.2**	
2006			**126.6**	**12.0**	
2007			**150.7**	**8.8**	
2008	189.6	26.7	**118.2**	**5.0**	*22.9*
2009	189.6	26.7	*136.2*	*18.3*	*24.6*
2010	189.6	26.7	*142.4*	*19.3*	*25.0*
2011	189.6	26.7	*148.1*	*20.2*	*25.4*
2012	189.6	26.7	*153.3*	*21.0*	*25.7*
2013	189.6	26.7	*158.0*	*21.7*	*25.9*
2014	189.6	26.7	*162.2*	*22.4*	*26.1*
2015	189.6	26.7	*165.9*	*23.0*	*26.3*
2020	189.6	26.7	*178.7*	*25.0*	*26.6*
2030	189.6	26.7	*187.5*	*26.3*	*26.7*

See note to table 1 above. The reference point rule used by NOAA for the Tanner Crab is $H_t = H_{\text{MSY}}\left(X_t/X_{\text{MSY}} - 0.1\right)/0.9$.

Table 3: Projected Harvests and Stocks (millions lbs)
Bering Strait Snow Crab

	X_{MSY}	H_{MSY}	X_t	H_t	$g_{\text{L}}\left(X_t\right)$
2005			**188.0**	**46.9**	
2006			**218.0**	**49.4**	
2007			**218.0**	**78.6**	
2008	317.7	101.9	**241.0**	**69.5**	*96.0*
2009	317.7	101.9	*267.5*	*85.8*	*99.4*
2010	317.7	101.9	*281.0*	*90.1*	*100.5*
2011	317.7	101.9	*291.4*	*93.5*	*101.2*
2012	317.7	101.9	*299.2*	*96.0*	*101.6*
2013	317.7	101.9	*304.8*	*97.8*	*101.7*
2014	317.7	101.9	*308.7*	*99.0*	*101.8*
2015	317.7	101.9	*311.5*	*99.9*	*101.9*
2020	317.7	101.9	*316.8*	*101.6*	*101.9*
2030	317.7	101.9	*317.7*	*101.9*	*101.9*

See note to table 1 above. The reference point rule used by NOAA for the Snow Crab is $H_t = H_{\text{MSY}}\left(X_t/X_{\text{MSY}} - 0.1\right)/0.9$.

Note that while the King crab stocks were above their maximum sustainable yield biomass in 2008, the Snow and Tanner crab stocks were below their corresponding MSY biomass levels. For these two crab stocks, the reference point transition function applied by NOAA prescribes a harvesting rate that approaches the MSY from below. As time passes the projected harvest appoaches H_{MSY}, with the H_{MSY} being reached by 2030 for both the Snow and Tanner crab stocks. This demonstrates the importance of knowing where the current stock level is in relation to the maximum sustainable yield stock level is when projecting the harvesting rates into the future.

3.2 Valuation.

Tables 4 and 5 below summarize the economic data for the entire crab fishery. Note that we impute a gasoline cost using average fuel prices in Alaska. The Red King crab, Tanner crab and Snow crab fisheries exhibit substantial variation in revenue over the period. This is more due to volatility in the price received per pound of crab over the time period rather than from changes in volume of catch.

Table 4: Revenue and Costs for Alaskan Crab Fisheries

Year	revenue mil $	labor mil $	bait mil $	observer costs mil $	gas mil $	avg. return mil $
			Bristol Bay Red King Crab			
1998	42.38	9.38	1.17	0.03	n/a	n/a
2001	45.57	9.83	0.96	0.16	n/a	n/a
2004	78.35	17.31	1.32	0.02	n/a	n/a
2005	84.87	11.16	0.75	0.00	25.70	47.26
2006	60.95	8.17	0.59	0.00	23.08	29.11
2007	87.84	11.03	0.77	0.00	30.94	45.10
2008	96.97	13.15	0.98	0.00	48.04	34.80
			Bering Strait Snow Crab			
1998	150.13	33.77	4.28	0.12	n/a	n/a
2001	39.27	7.91	1.33	0.05	n/a	n/a
2004	54.33	11.68	1.18	0.06	n/a	n/a
2005	46.24	10.00	0.94	0.04	30.88	4.38
2006	39.90	5.66	0.54	0.00	38.25	-4.55
2007	56.74	8.34	0.45	0.00	31.58	16.37
2008	98.41	14.60	0.65	0.00	68.82	14.35
			Bering Strait Tanner Crab			
1998	–	–	–	n/a	n/a	n/a
2001	0.03	0.00	0.00	n/a	n/a	n/a
2004	–	–	–	n/a	n/a	n/a
2005	0.44	0.05	0.01	0.01	0.09	0.35
2006	1.44	0.22	0.02	0.00	0.54	0.90
2007	3.99	0.58	0.08	0.00	0.76	3.23
2008	4.04	2.73	0.12	0.00	1.09	2.95

"Observer costs" refer to the costs associated with monitoring of fishing practices by the regulator. Data obtained from the 2010 SAFE.

Table 5: Costs for all Alaskan Crab Fisheries

	Taxes mil $	Co-op fees mil $	Freight mil $	Storage mil $	Gear mil $	Pots mil $	Repairs mil $	Capital Invst. mil $
Additional Costs: All Fisheries								
1998	4.19	n/a	0.01	1.53	3.49	3.97	0.02	22.47
2001	1.94	n/a	n/a	1.23	1.80	0.96	0.02	13.26
2004	2.84	n/a	n/a	1.53	1.93	0.95	0.02	17.88
2005	5.27	0.28	0.00	0.96	0.84	0.69	0.00	0.55
2006	5.92	0.47	0.00	0.54	0.91	0.54	0.00	0.55
2007	9.50	0.82	0.00	0.59	0.73	0.37	0.00	1.87
2008	13.46	0.56	0.00	0.87	1.28	0.53	0.00	0.68

Data obtained from the 2010 SAFE. Note that 2008 refers to the 2008-2009 fishing season.

We thus have data on the average profitability of the crab fishery. As stated under the previous subsection, under the assumption of perfect competition, constant marginal cost, and no search costs, average profit is equal to marginal profit per unit. We can use the projected growth in the harvest - given in tables 1 to 3 - to back out an estimate of the profitability of the fishery. We assume all costs per unit and revenues per unit are the same as in the 2008 season. Then using the discount factor δ, we can calculate the net present value of the fishery. In doing so, we omit taxes from the fixed costs. That is, the valuation of the crab stock at time 0 is given by

$$V_0 = \sum_{t=1}^{\infty} \left(\frac{1}{1+\delta} \right)^t \left(H_t^{\text{KING}} \pi^{\text{KING}} + H_t^{\text{TANNER}} \pi^{\text{TANNER}} + H_t^{\text{SNOW}} \pi^{\text{SNOW}} - C \right)$$

where π^{KING} denotes the per-lb profit of the red king crab in 2008, and H_t^{KING} denotes the projected harvest rates of the king crab at time t, etc., and C denotes the costs given in table 5 that cannot be attributed to a single crab stock.

Note the similarity to the valuation equation (1) above. Here, however, we are working in discrete time. The discount factor $(1+\delta)^{-t}$ corresponds to $e^{-\delta t}$ in (1). In our example only the King Crab stock is being harvested at the MSY level; for the remaining stocks, the annual harvest increases over time as the stock level is built back up to the MSY level. But because we have the harvesting policy of the regulator, we can project future harvests H_t once we have made an assumption about the growth function $g(X_t)$. Last, because of our assumptions on market and firm structure, the marginal profit per unit is expected to remain constant. That is $p(H_t) - c(X_t, H_t)$ in (1) is constant.

Table six gives the valuations for various discount factors δ.

Table 6: Bering Sea Crabs
Valuation in current $ (mil)

$\delta = 0.04$	$\delta = 0.07$	$\delta = 0.1$
1,704.299	973.605	678.756

Table 6 demonstrates that the choice of discount rate has a large impact on the value of the asset. Increasing the discount rate from 4% to 10% more than halves the value. As discussed earlier, in practice the determination of the appropriate discount rate is an important task. Reporting valuations for a range of discount rates may also be desirable in order to reflect the sensitivity of the analysis to key model parameters. To put the range of valuations in context, the total revenue from the Alaskan "Fisheries, Forestry and related" NAICS industry was $316 million in 2008.

Note that capital costs have been excluded from all calculations. A final step would be to subtract the value of the capital used in extraction - namely the fleet of boats - from the stated value of the fishery above. As this data is lacking, we do not take his final step, and instead treat the figures given in the table above as a joint valuation of the fishery and fleets together.

4 Conclusion.

This paper summarizes the extant literature on bioeconomics. Our specific focus is outlining the necessary assumptions that permit valuation of near-market assets based on the average profits obtained from the asset. It is shown that for resources that have a single owner, perfect competition, constant marginal cost, and independence of costs from asset abundance are sufficient assumptions on the economic side for valuation. On the biological side, it is necessary to make an assumption of the growth function of the asset.

We apply the valuation methods to a specific US crab fishery, but the methods can be applied to a wider range of assets that grow endogenously, such as other fishery species, forestry, agriculture and horticulture. We found that the valuation of the crab fishery is not robust to changes in the discount rate.

5 Appendix: Classic Hotelling Valuation

In the conventional Hotelling framework the amount of the asset X_t is finite. Hence our maximization constraint becomes

$$\frac{\partial X_t}{\partial t} = -H_t, \quad \int_0^\infty H_t dt = X_0$$

Now for the price-taker with constant marginal cost, we have $\pi(H_t, X_t) = (p_t - c_t) H_t$, where c_t is a constant marginal cost. The solution is as follows. The present value Hamiltonian is

$$\mathbf{L}(H_t, X_t, \psi) = e^{-\delta t}\pi(H_t, X_t) - \lambda H_t$$

Then the FOCS are

$$\begin{aligned} \frac{\partial \mathbf{L}(H_t, X_t, \psi)}{\partial H_t} &= 0 \Longrightarrow e^{-\delta t}(p_t - c_t) = \lambda_t \\ \frac{\partial \mathbf{L}(H_t, X_t, \psi)}{\partial X_t} &= -\dot{\lambda}_t \Longrightarrow \dot{\lambda}_t = 0 \end{aligned}$$

so $\lambda_t = \lambda$ is constant. Solving out we have $e^{-\delta t}(p_t - c_t) = \lambda$, which implies that $p_t - c_t = e^{\delta t}(p_0 - c_0)$. Then the value asset is

$$V_0 = \int_0^\infty e^{-\delta t} e^{\delta t}(p_0 - c_0) H_t dt = (p_0 - c_0)\int_0^\infty H_t dt = (p_0 - c_0) X_0$$

since $\int_0^\infty H_t dt = X_0$, which is the classic Hotelling valuation principle.

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