

# A Multivariate Approach to Seasonal Adjustment\*

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## Abstract

This paper suggests a new semi-parametric multivariate approach to seasonal adjustment. The primary innovation is to use a large dimensional factor model of cross section dependence to estimate the trend component in the seasonal decomposition of each time series. Because the trend component is specified to capture covariation between the time series, common changes in the level of the time series are accommodated in the trend, and not in the seasonal component, of the decomposition. The seasonal components are thus less prone to distortion resulting from severe business cycle fluctuations than univariate filter-based seasonal adjustment methods. We illustrate these points this using a dataset that spans the 2007-2009 recession in the US.

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# 1 Introduction

The potential distortionary effects of the 2007-2009 recession (or “Great Recession”) on seasonally-adjusted data has received significant academic, political and media attention (The White House, 2013, Bialik, 2012; Goldstein, 2013). If not properly accounted for in a seasonal adjustment method, the severe decline in US economic activity over late 2008 and early 2009 can be interpreted as a change in seasonal patterns<sup>1</sup>, leading seasonally adjusted data to be spuriously strong in the first and fourth quarters, and spuriously weak in the second and third quarters (see, e.g., Institute of Supply Management, 2012; Federal Reserve Board, 2011; Feroli, 2012). To mitigate the effect of the recession, much of the seasonally-adjusted data published by statistical agencies were subjected to a so-called “intervention”, whereby the default seasonal adjustment method is manually altered by the analyst (see, e.g., Federal Reserve Board, 2011; Kropf and Hudson 2012). Although various commentators speculate that published economic data was distorted in the wake of the recession (Furth and Sherk, 2013), studies suggest that main economic indicators such as employment and GDP have not been distorted due to the appropriate application of analyst interventions (Kropf and Hudson 2012; Evans and Tiller, 2012; Macroeconomic Advisors, 2012).

Motivated by this ongoing debate, in this paper we consider a new “multivariate” approach to seasonal adjustment that is less prone to the potentially distortionary impacts of severe business cycle fluctuations. Conventional seasonal adjustment factors are typically estimated using the past history of an individual time series. Popular examples of these “univariate” approaches to seasonal adjustment include filter-based methods, such as X-11, X-11-ARIMA, X-12-ARIMA and X-13-ARIMA-SEATS, as well as model-based methods such as TRAMO-SEATS. Under the “multivariate” approach a large cross section of time series are modelled jointly for the purpose of estimating seasonal effects for each individual time series.<sup>2</sup>

The multivariate approach incorporates many of the elements of the univariate filter-based methods. However, the primary difference between the multivariate and the univariate approaches is the conceptual treatment of the stochastic trend in the modeling framework. Because many time series are non-stationary or exhibit substantial persistency, time series must first be de-trended before estimating the seasonal components used to adjust the data. Under a filter-based univariate approach the trend is estimated based on the individual time

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<sup>1</sup>Real GDP fell more than 7 percent at an annual rate over the fourth quarter of 2008 and the first quarter of 2009, and total nonfarm payroll employment plunged by more than 4 million jobs from September 2008 to March 2009.

<sup>2</sup>“Seasonal effects” or “seasonal components” are often referred to as “seasonal adjustment factors”. We use the term “factors” in reference to the common factor model embedded in the multivariate seasonal decomposition.

series under consideration, typically by smoothing the time series using a moving average filter. (See figure 1 below for an example of a basic smoothed trend.) Under the multivariate approach, the fitted stochastic trend captures both the long term variation in the time series as well as the high frequency covariation between the cross section of time series. This is achieved using a flexible parametric model of cross section dependence, specifically an approximate factor model with non-stationary common factors and stationary idiosyncratic components (Chamberlain and Rothschild, 1983; Bai, 2003; 2004). Apart from the nature of the trend estimation, the seasonal components of the time series are estimated using the same nonparametric seasonal filters that are used in the filter-based methods. Specifically, the seasonal components are obtained by taking moving averages of adjacent months or quarters over an individual de-trended time series.

The primary advantage of the multivariate approach is that common, abrupt changes in the levels of the time series do not distort the seasonal patterns generated by the seasonal adjustment model. When a univariate smoothing filter is used to estimate the trend component, a sudden change in the level of a time series is graduated over many time periods, and consequently the de-trended time series will exhibit sustained deviations from zero either side of the turning points in the time series.<sup>3</sup> This can generate spurious changes in the seasonal patterns implied by the seasonal filters, unless the model is adjusted using an “intervention” built into the seasonal adjustment procedure (see section 3.3 below). In contrast, because the approximate factor model is explicitly designed to capture covariation, under the multivariate approach a common change in the levels of the time series is accommodated in the fitted trend component - and not the seasonal component - of the model. Because recessions coincide with a decline in a large cross section of time series, this means that the multivariate approach is better at seasonally adjusting data that is subject to the sharp turning points associated with the peaks and troughs of severe recessions.

The seasonal components obtained from the multivariate approach should be less distorted by the 2007-2009 recession than the seasonal components obtained from a univariate approach. To explore the performance of the multivariate and univariate seasonal adjustment methods we consider seasonally adjusting a panel of disaggregate nominal imports that span the 2007-2009 recession. The fall in economic activity in the fourth quarter of 2008 and the first quarter of 2009 was particularly severe in the trade sector (Bridgman, 2013). For example, nominal crude oil imports fell by 76% between July 2008 and February 2009.

We rely on two evaluation criteria for assessing the seasonal adjustment methods. First, we directly test whether the recession has distorted the fitted seasonal components of each method. Most of the import series over the late 2008 to early 2009 period. If that decline

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<sup>3</sup>Economists and Statisticians have long been aware of this potential deficiency of smoothing filters. See, e.g., Macaulay (1931).

is not fully accommodated in the fitted trend, then part of the decline will be pushed into the fitted seasonal components, causing the seasonal components in quarter one or four to be smaller (and the seasonal components in quarter two or three to be larger) in the years preceding, during and following the recession. Our test is based on establishing the statistical significance of these patterns. Second, we consider revisions to seasonal components. As illustrated by Fixler and Grimm (2002) and Fixler, Grimm and Lee (2003), revisions to seasonal adjustment factors can be a large source of revisions to seasonally-adjusted national economic data. Hence, a seasonal adjustment procedure that yields stable seasonal components but has large revisions may not be that useful in practice. To explore the revision performance of the two methods, we estimate the seasonal components over successive years, revising previously estimated seasonal components as calendar time progresses. Evaluation of the two approaches is based on the magnitude of the revisions to the seasonal components, with smaller revisions being preferred to larger revisions.

To preview our results, we find that the multivariate approach fares better when evaluated by either criterion. There is little evidence of a pervasive change in the multivariate model seasonal components as a result of the recession. In contrast, there is substantial evidence of pervasive changes in the univariate seasonal components coinciding with the recession. In addition, the revisions to multivariate seasonal components are smaller (in absolute magnitude) than the revisions to the univariate seasonal components over the 1998 to 2011 period.

The remainder of the paper is organized as follows. In section two we introduce the multivariate seasonal adjustment method. In section three we compare the univariate seasonal adjustment to the multivariate approach, highlighting some of the key advantages of the multivariate method. Section four compares the performance of a filter-based univariate approach to the multivariate method using the nominal import data. We then conclude. For readers who may not be familiar with the specifics of the X-11 and similar procedures, the appendix contains an outline of the non-parameteric filter-based seasonal adjustment methods. Throughout  $\text{tr}(A)$  denotes the trace of a square matrix  $A$ .

## 2 Multivariate Seasonal Adjustment

Our procedure is based on an additive decomposition of the form

$$x_{i,t} = c_{i,t} + s_{i,t} + u_{i,t}, \tag{1}$$

where  $t$  indexes the time period and  $i = 1, \dots, n$  indexes the cross sections in the panel dataset. The observable variable  $x_{i,t}$  is decomposed into three components:  $c_{i,t}$  denotes the “trend” (or “trend-cycle”) component,  $s_{i,t}$  is the “seasonal” component, and  $u_{i,t}$  is the

“irregular” component.<sup>4,5</sup> The fitted trend component captures long-term variation in the level of an individual time series, including any non-stationarity. The seasonal component captures predictable patterns in the time series that are related to the time of year (annual cycles). Seasonally-adjusted time series are obtained by subtracting the estimated seasonal components from the original time series. Finally, the irregular component acts as a residual, capturing variation not included in the trend or the seasonal components.<sup>6</sup>

The decomposition of time series into trend, seasonal and irregular components in (1) is similar to univariate modeling frameworks such as X-11. However, the primary difference between the multivariate approach and the univariate approach is the treatment of the trend component. In univariate procedures, the trend component is estimated using a nonparametric smoothing filter applied to an individual time series. (See section 3.1 below.) It therefore captures only the low-frequency variation in the time series, so that high frequency variation is pushed into the remaining seasonal and irregular components. Under the multivariate approach, the trend component will be fitted using a parametric model of covariation between a large number of time series. This permits it to capture both the long-term variation in the level of an individual time series as well as the high frequency covariation between the cross section of time series. Specifically, we use a factor model of the form

$$c_{i,t} = \sum_{j=1}^r f_{j,t} \lambda_{j,i}$$

The  $f_{j,t}$  are referred to as “common factors”, and  $\lambda_{j,i}$  are referred to as “factor loadings”.  $r$  denotes the number of common factors, assumed to be finite in our parametric context. Under the additional assumption that the irregular component is weakly dependent across time series (specifically we assume the the panel of irregular components obeys Assumption A below), the factor structure accounts for both the long-term variation in the level of a time series (persistence) as well as the high frequency covariation between time series (cross section dependence).

Because it explicitly accounts for high frequency covariation in the time series, the fitted trend can accommodate abrupt common changes in the level of the times series. (By “common”, we mean that the event affects a large proportion of the time series at approximately the same time. This idea is formalized in Assumption B below.) In section 3 we

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<sup>4</sup>The decomposition into a trend, seasonal and irregular component is conceptually similar to the filter-based seasonal adjustment methods, such as the X-11 family.

<sup>5</sup>The multiplicative decomposition involve multiplying (rather than adding) these three components together, and as such the additive decomposition can be obtained by taking the logarithm of the multiplicative model.

<sup>6</sup>Seasonal factor models such as (1) are uncommon in the extant seasonal adjustment literature. Camacho, Lorch and Perez-Quiros (2012) is a recent example, although the focus of the paper is not on seasonal adjustment.

explain how this feature gives the multivariate approach an advantage over the univariate approach with respect to the potential distortionary effects of the 2007-2009 recession on seasonally-adjusted data.

Given the estimate of the trend component, the seasonal components are then estimated using the same approach as used in the univariate filter-based procedure. Moving average filters over adjacent months are applied to each time series once the fitted trend has been removed. (See the Appendix for more details.) This part of the procedure is nonparametric, permitting the fitted seasonal components for a given month to change slowly over time.

Conventional factor models (without an explicit seasonal component) are commonly used in a variety of settings, such as finance (Ludvigson and Ng, 2005, 2007), macroeconomic forecasting (Stock and Watson, 2002b; Artis et al., 2005; Marcellino et al., 2003; and Forni et al., 2001), policy analysis (Bernanke and Boivin, 2003; Giannone et al., 2005a, 2005b; Favero et al., 2005; Stock and Watson, 2005; and Forni et al., 2003) and price measurement (Cristadoro et al., 2001; Reis and Watson, 2010). The properties of various estimators of the conventional factor model are well-established in the extant literature, such as Stock and Watson (2002a), Bai and Ng (2002), Bai (2003, 2004), and Forni et al. (2000, 2004, 2005).

We use the principal components estimator of the factor model in the estimation of (1). In the appendix we give an overview of the principal components estimator and the standard assumptions placed on the model components for identification. Readers who are unfamiliar with the estimator may wish to read the appendix before tackling the subsection below.

## 2.1 Seasonal factor model estimation

Let  $\mathbf{X} = (x_{i,t})$  denote a  $T \times n$  matrix of the panel data. In matrix notation, the factor model is

$$\mathbf{X} = \mathbf{F}\mathbf{\Lambda}' + \mathbf{S} + \mathbf{u} \tag{2}$$

where  $\mathbf{F}$  is a  $T \times m$  matrix of stochastic factors,  $\mathbf{\Lambda}$  a  $n \times m$  matrix of factor loadings,  $\mathbf{S}$  is a  $T \times n$  matrix of seasonal components, and  $\mathbf{u}$  is a  $T \times n$  matrix of idiosyncratic components.  $\mathbf{F}\mathbf{\Lambda}'$  is a matrix of the common components.

If seasonality is pervasive across time series in the panel (i.e., if the majority of the times series in the panel exhibit seasonality), simply applying the principal component estimator to  $\mathbf{X}$  would yield estimates of stochastic common factors that embody both the non-seasonal trending component to  $\mathbf{X}$  as well as seasonal components. Hence, in the equation above, we make a conceptual distinction between the non-seasonal stochastic factors  $\mathbf{F}$  and the seasonal components  $\mathbf{S}$ , that will be reflected in the estimation of these components.

There are several possible methods to distinguish between the non-seasonal and seasonal common factors. For example, Camacho, Lorcha and Perez-Quiros (2012) use a parametric

approach to estimate both the factors and the seasonal components. In this paper we will use a semi-parametric approach that permits the seasonal components to vary over time. We use an initial estimate of the seasonals  $\mathbf{S}$  before implementing the recursive estimation of trends and seasonal components. Specifically, we do the following:

- (i) For some initial estimate of the seasonal components  $\hat{\mathbf{S}}_0$ , estimate the common factors  $\hat{\mathbf{F}}_0$  to  $\mathbf{Y} - \hat{\mathbf{S}}_0$ . In the empirical part of this paper, our initial estimate of the seasonal components will be the normalized average growth rates of the time series in each month (The monthly averages are normalized to be mean zero).
- (ii) Remove the fitted stochastic trends from  $\mathbf{X}$ , i.e.,  $\mathbf{X} - \hat{\mathbf{C}}$ , where

$$\hat{\mathbf{C}} = \hat{\mathbf{F}}_0 \left( \hat{\mathbf{F}}_0' \hat{\mathbf{F}}_0 \right)^{-1} \hat{\mathbf{F}}_0' \mathbf{X} = \hat{\mathbf{F}}_0 \hat{\mathbf{\Lambda}}_0'.$$

- (iii) Estimate new seasonal components  $\hat{\mathbf{S}}_1$  from the de-trended series. Seasonals are estimated using the same filters as in the univariate X-11 procedure, i.e., by applying seasonal filters (moving averages over adjacent months or quarters). Proceed back to step 1, using  $\hat{\mathbf{S}}_1$  in place of  $\hat{\mathbf{S}}_0$ .
- (iv) Recursively update the estimated stochastic trends and the seasonal components until convergence occurs.

The recursive procedure outlined above only differs from the conventional X-11 univariate approach in how the stochastic trend component is estimated. It is subject to many of the same model specification decisions, such as the nature of the seasonal filter. As in X-11, the seasonal factors are estimated using moving averages on the de-trended data,  $\mathbf{X} - \hat{\mathbf{C}}$ .

Because the seasonal filter is a centered moving average, complications can arise as the filters reach the endpoints of the sample. Throughout, we will rely on asymmetric trend filters that are computed using only the data available up to the time of the final observation in the times series.<sup>7</sup>

## 2.2 Updating Seasonal Components

As time passes more data become available. The model used to estimate the stochastic trend is parametric, and as additional data are used in the estimation, the estimated trends can change over the entire time series dimension of the sample. This is a potentially substantial

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<sup>7</sup>That is, if we are estimating the seasonal components up to an including March 2009, we use all available data up to and including March 2009. As discussed in section 3.1, another option is to forecast each time series. We leave the development of the necessary panel forecasting techniques to future research.

source of variation in different vintages of the seasonal components. In effect, the entire history of seasonal components could be revised as new data became available.

In contrast, the smoothing filters used in the univariate filter-based methods estimate the trend using a limited amount of time series data (see section 3.1 below), typically between one and two years. This means that the univariate trend for a given month or quarter is not revised after one year has passed. This gives the nonparametric filter-based method a significant advantage in terms of reducing revisions to seasonal components.

We therefore suggest that as new data become available as time passes, the fitted trend component be held fixed, and that only the trend component for the new data be based on the most recently updated model. For example suppose that the seasonal components are estimated every year. The seasonal components for the 2008 year would be estimated using data available up until the end of 2008. As the data for 2009 become available, the seasonal components for 2009 would be estimated using the data available up until the end of 2009, while the trend component for 2008 would not be updated.

In addition, to permit the parameters in the model to change slowly over time, we suggest that the factor model be estimated using a “rolling window” methodology, wherein the time series dimension of the sample used in estimation is held fixed. Returning to the example above, the seasonal components for the 2008 year could be estimated from data spanning 1999 to 2008 (corresponding to a 10 year window), and the seasonal components for the 2009 year could be estimated from data spanning 2000 to 2009, and so on.

To formalize this estimation method, we introduce the following notation. Let

$$\mathbf{X}_{(t_1, t_2)} = \{X_s; t_1 \leq s \leq t_2\}$$

denote the observed panel from time  $t_1$  to  $t_2$ , so that  $\mathbf{X}_{(t_1, t_2)}$  is a  $(t_2 - t_1) \times n$  matrix. We then let

$$\hat{\mathbf{C}}_{(t_1, t_2)} = \left\{ \hat{C}_{s, (t_1, t_2)}; t_1 \leq s \leq t_2 \right\}$$

denote the  $(t_2 - t_1) \times n$  matrix of stochastic trends estimated using  $\mathbf{X}_{(t_1, t_2)}$ , where  $\hat{C}_{s, (t_1, t_2)}$  denotes an  $n \times 1$  vector of the cross section of fitted trends at time  $s$ . Let  $freq$  denotes the sub-annual sampling frequency of the data (e.g.,  $freq = 12$  for monthly data). Then our proposed estimation method is of the form

$$\left\{ \tilde{C}_s \right\}_{s=1}^T = \left\{ \hat{C}_{s, (\bar{t}-S, \bar{t})} \right\}_{s=1}^T, \quad \bar{t} < s \leq \bar{t}$$

and

$$\bar{t} = \left\lceil \frac{s}{freq} \right\rceil \times freq, \quad \underline{t} = \left\lfloor \frac{s}{freq} \right\rfloor \times freq,$$



and where  $S$  denotes the fixed time series dimension of the sample used in estimation (e.g., to base estimation on ten years of data,  $S = 120$  for monthly data),  $\lceil \cdot \rceil$  denotes the smallest integer greater than or equal to the argument, and  $\lfloor \cdot \rfloor$  denotes the largest integer less than or equal to the argument.

### 3 Comparison of Multivariate and Univariate Seasonal Adjustment

In this section we compare the multiplicative seasonal adjustment method outlined in the previous section to a conventional filter-based univariate procedure similar to the X-11 family of seasonal adjustment methods. We begin with a brief overview of the filter-based methods, before moving on to discuss the advantages of the multivariate approach in dealing with severe business cycle fluctuations. In the final subsection we outline some of the interventions that can be used in the X-11 module to mitigate the distortionary effects, and we will argue that the advantages of the multivariate approach are likely to be greater for timely seasonal adjustment, such as concurrent seasonal adjustment.

#### 3.1 Univariate Filter-based Seasonal Adjustment

The canonical additive decomposition of a time-series  $x_t$  for filter-based seasonal adjustment is as follows.

$$x_t = c_t + s_t + u_t, \tag{3}$$

where  $c_t$  denotes the “trend” (or “trend-cycle”) component,  $s_t$  denotes the “seasonal” component, and  $u_t$  denotes the “irregular” component. In X-11 (and subsequent iterations thereof, such as X-12 and X-13) the trend and seasonal components are estimated using moving average filters. In the first pass, the trend component is estimated using a smoothing filter. The trend is then subtracted away from the time series, and the seasonal components are estimated by applying a second moving average over adjacent months. The seasonal filter therefore spans a limited number of years, permitting the fitted seasonal components to vary slowly over time. The seasonal components are normalized to be mean zero within a calendar year.

Because these moving averages are typically centered moving averages, complications can arise as the filters reach the endpoints of the sample. Either asymmetric filters can be used (as in X-11), or the unavailable series can be forecast (using the ARIMA module in X-11-ARIMA or X-12-ARIMA). Throughout, we will rely on asymmetric trend filters that are computed using only the data available at the time of the final observation in

the times series.<sup>8</sup> Similarly, for both the multivariate and the univariate methods, we use asymmetric seasonal filters at the end points of the sample. This approach corresponds to “concurrent” seasonal adjustment, as opposed to “factor projected” seasonal adjustment. Under the latter approach, the seasonal components for the a given period are initially computed a few periods after the close of the reference period. In the interim, the seasonal component(s) from previous periods are used. By delaying the computation of the seasonal components, the analyst has more data with which to fit the smoothed trend.

### 3.2 Business Cycle Fluctuations

Both the nonparametric univariate and the semi-parameteric multivariate models can accommodate business cycle fluctuations in the “trend” component of the respective models. However, the smoothing filter - used to estimate the trend component under the univariate approach - graduates high frequency variation in the data over several periods. The de-trended time series that is subsequently used to estimate the seasonal components will exhibit persistent deviations from zero around these turning point(s), and this will distort the seasonal pattern in the de-trended data.<sup>9</sup>

As an example, nominal crude oil imports experienced a particularly acute fall during the recession, declining by 76% between July 2008 and February 2009. This fall is too large and sudden to be accommodated in a simple moving average. Figure 1 depicts a basic  $2 \times 12$  moving average (a 13-month moving average) estimated at two different points in time: March 2008 and March 2009. The vintages are particularly instructive because the trough in nominal imports occurs in February 2009, and the brief run up in imports over the summer of 2008 has not yet occurred in March 2008. The latter estimate begins to decline in early 2008, and reaches a nadir in mid 2009, thus smoothing the period of decline over more than a year. Consequently, the de-trended series exhibited in figure 2 has a positive and persistent deviation from zero over March to October 2008, and a persistent negative deviation from November 2008 to June 2009.

Figure 3 exhibits the seasonal components obtained by applying a standard  $3 \times 9$  seasonal filter to the de-trended series in figure 2. The  $3 \times 9$  filter spans eleven years of data. The

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<sup>8</sup>We use asymmetric filters as preliminary analysis showed that automated ARIMA forecasts performed poorly over the recession. Because the univariate approaches rely more heavily on future data (specifically for trend estimation), it would seem that using ARIMA forecasts would disadvantage the univariate approach more than the multivariate approach. In addition, we use asymmetric filters for the multivariate approach.

<sup>9</sup>The seasonal filters use a limited number of observations to estimate the seasonal component for a given month in a given year. For example, a  $3 \times 3$  filter uses 5 years of data. One potential way to limit the effect of an irregularity is to lengthen the span of the seasonal filter, with the intention of averaging away the irregularity in a longer span of data. In what follows, we use a  $3 \times 9$  filter (spanning 11 years) to demonstrate that this approach is does not sufficiently ameliorate the distortions caused by the recession.

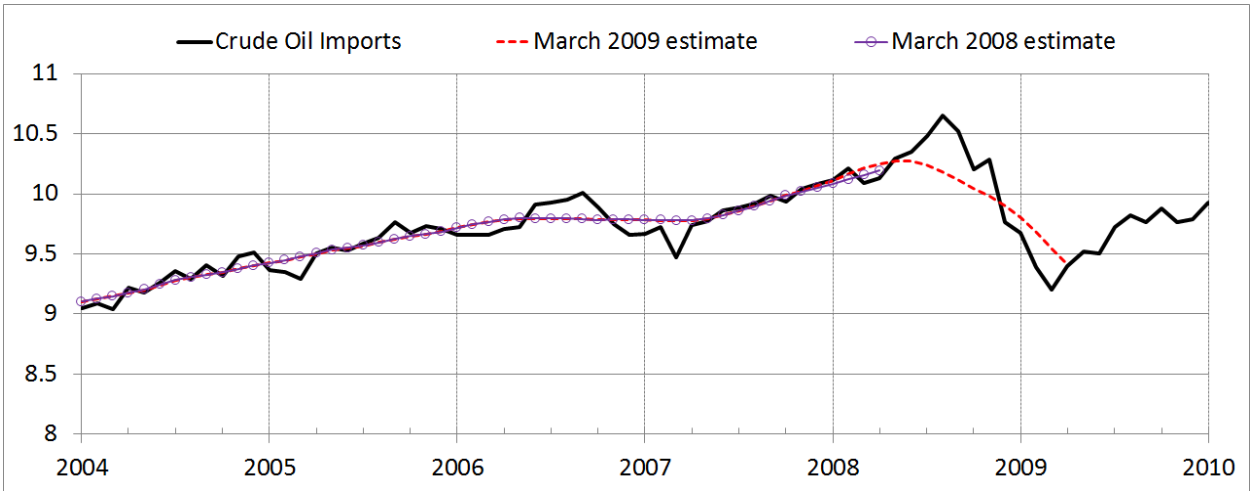


Figure 1: Log crude oil imports and  $2 \times 12$  trend, March 2008 and March 2009 vintage estimates.

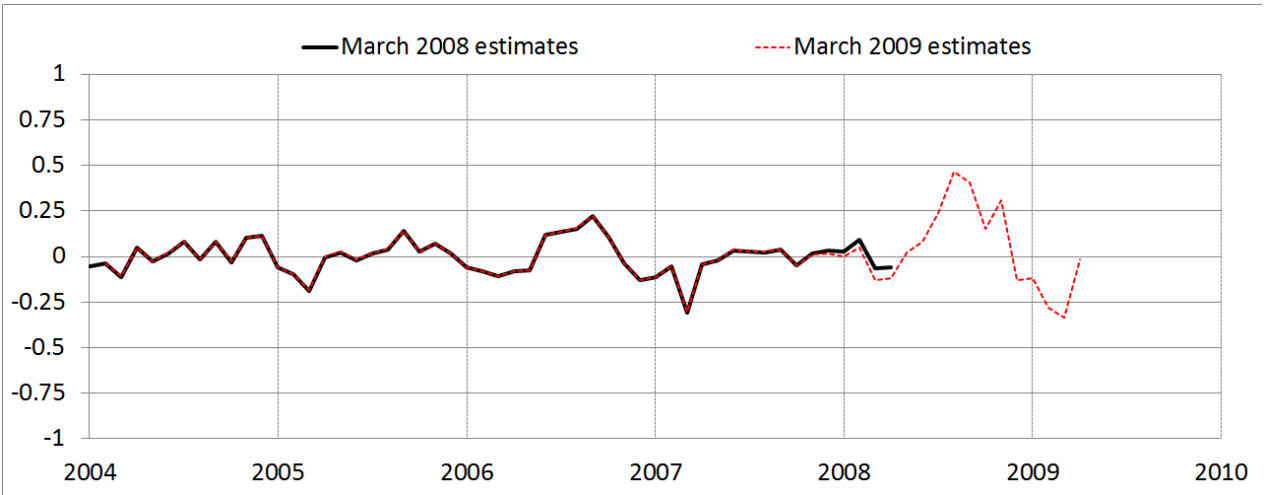


Figure 2: Vintage estimates of de-trended log crude oil imports. Trend component estimated using the  $2 \times 12$  filter.

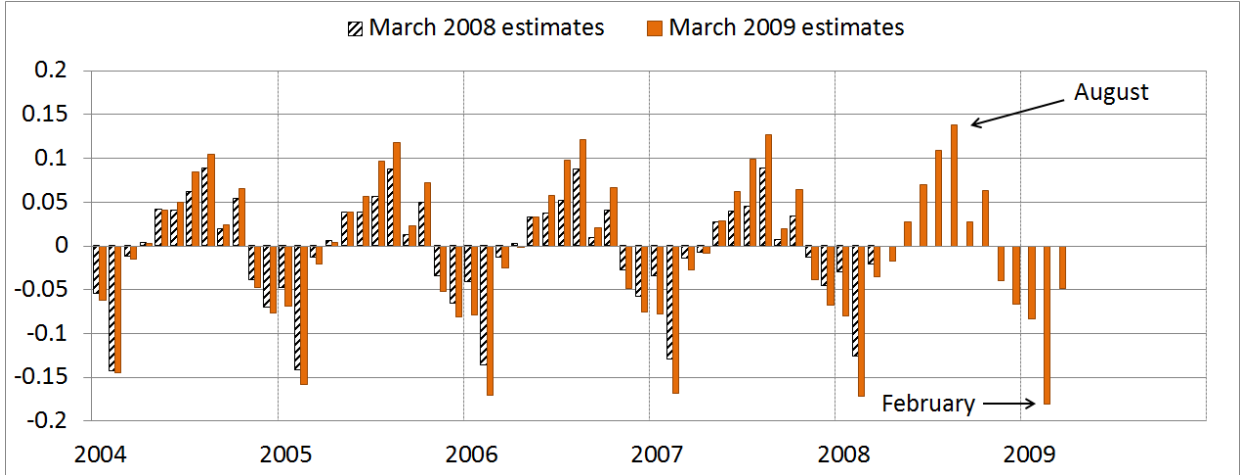


Figure 3: Vintage estimates of seasonal components to log crude oil imports. Trend component estimated using the  $2 \times 12$  trend filter; seasonal component estimated using the  $3 \times 9$  seasonal filter.

seasonal pattern in the March 2008 estimates (before the severe downturn beginning in August 2008) shows that imports usually increase over the summer months before falling over winter. Imports are lowest in February, although it appears that February imports were getting slightly larger over the 2004 to 2008 period (note that the bars get smaller in magnitude for this period).

The seasonal pattern is exacerbated in the March 2009 vintage: All seasonal components grow larger in magnitude once the additional year of data is included. This additional year contains the recessionary fall in nominal crude oil imports. As shown in figure 2 above, detrended imports reach a peak in August 2008 and a trough in February 2009. Correspondingly, the March 2009 vintage seasonal components for February become more negative (relative to the March 2008 vintage), and the March 2009 vintage seasonals for August become more positive (relative to the March 2008 vintage). From this we can safely conclude that the recession has distorted the seasonal components.

Note that for the March 2009 vintage estimate, the seasonal components grow larger in magnitude over the 2004 to 2009 period. This is also indicative of the recession distorting the seasonal components. The  $3 \times 9$  filter uses five years of data either side of the reference month (e.g., the seasonal component for February 2004 depends on the de-trended observation for February 2009), and hence the recession begins to affect the seasonal components from late 2003 onwards. This is made more clear in figure 4 below, in which we plot the March 2009 vintage estimates over a longer time period. The February seasonal component decreases

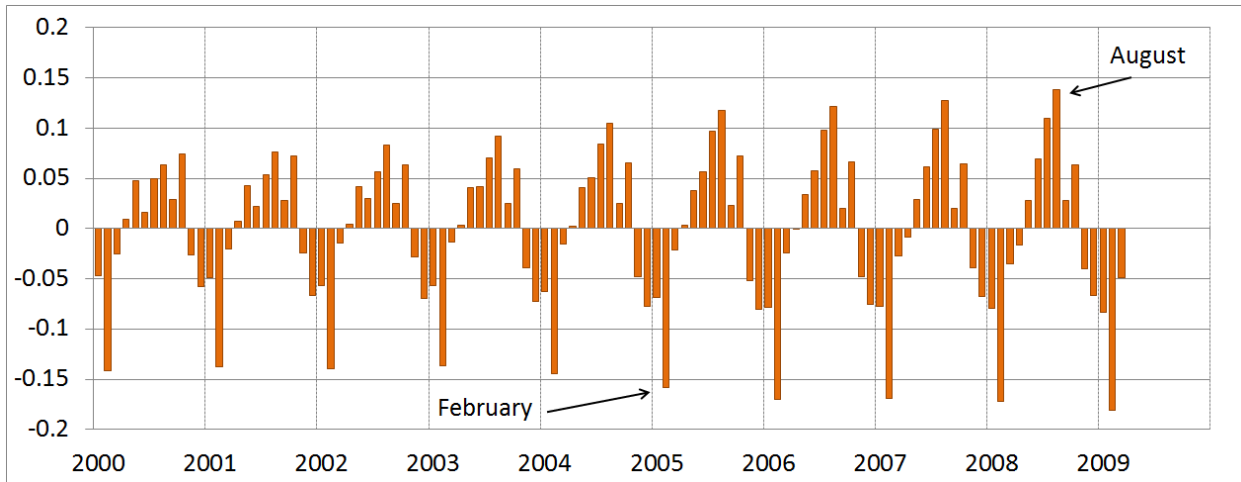


Figure 4: Seasonal components to log crude oil imports. Trend component estimated using the  $2 \times 12$  trend filter; seasonal component estimated using the  $3 \times 9$  seasonal filter. March 2009 vintage estimates.

from around -14% in the first half of the 2000 decade to -18% in 2009.<sup>10</sup> In the empirical evaluation will will exploit this pattern when testing for distortions in seasonal components.

The distortion in the seasonal components arises because the decline in nominal imports is too steep for the smoothing filter. This deficiency in the simple moving average filter has long been identified (see, e.g., Henderson, 1916; Macaulay, 1931; Musgrave, Shishkin and Young, 1967), and hence the X-11 family of procedures include alternative smoothing filters - such as the Henderson filter - that can better track inflections and turning points. We discuss this filter in more detail next.

### 3.2.1 Henderson Trend Filters

The Henderson (1916) filter minimizes the sum of squares of the third difference of the moving average series. This permits the fitted trend to track a local cubic polynomial, which means it can follow smooth inflections in the time series better than a simple moving average (such as the  $2 \times 12$  trend filter depicted in figure 1). For this reason it is favored as a method for estimating the low frequency variation in economic time series.<sup>11</sup>

Two caveats apply to the Henderson filter. First, like all smoothing filters, discontinuities

<sup>10</sup>Because the seasonal decomposition is additive and the import series has been logged, the seasonal components closely correspond to percentage deviations in the level of the nominal import series.

<sup>11</sup>Musgrave, Shishkin and Young (1967) advocate the use of the filter in their review of the X-11 procedure. The Australian Bureau of Statistics use the filter as a final estimate of the trend (Australian Bureau of Statistics, 2005).

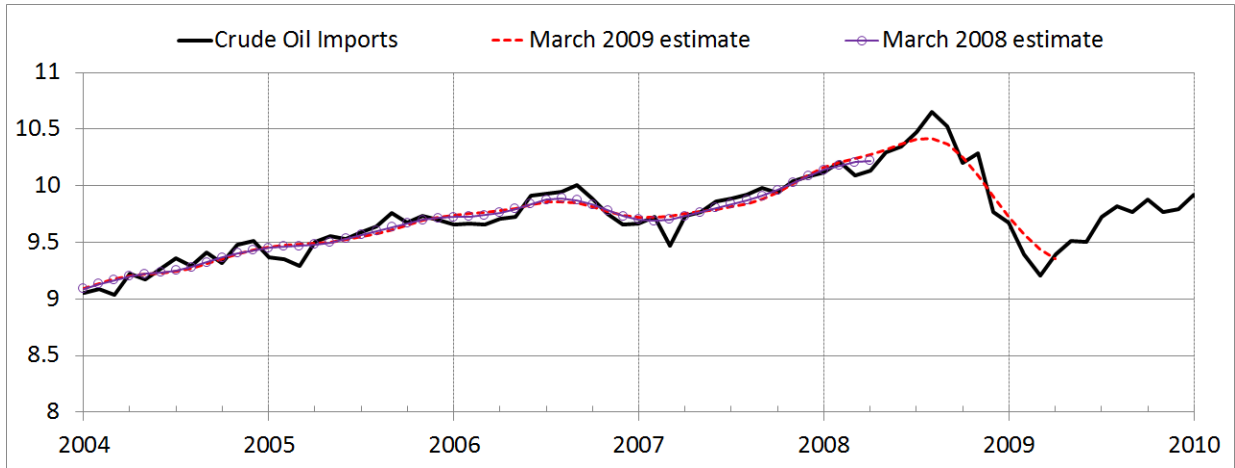


Figure 5: Log crude oil imports and 13 term Henderson trend, March 2008 and March 2009 vintage estimates.

in the slope of the original time series will be graduated over the span of the filter. This means that the abrupt changes in the level of the original time series will not be accommodated in the trend. Second, the filter dampens annual cycles in the time series, and therefore should not be used to estimate the trend component in non-seasonally-adjusted data (see, for example, chapter 5 of Australian Bureau of Statistics, 2005). For this reason, many seasonal adjustment procedures permit the Henderson filter to be used as a “second pass” estimate of the trend, after an initial first pass seasonal adjustment has been made. This first pass adjustment is typically made using a simple moving average method (see the Appendix for a step-by-step guide to this procedure). Thus any distortion in the first pass seasonal components will have a second order effect on the second pass estimates.

In figure 5 we depict the 13 term Henderson trend for crude oil over the same time period as in figure 1. It does a much better job at tracking the decline: For example, the peak in the trend coincides with the peak in the time series. However there is still some graduation of the decline. In particular, the slope of the smoothed trend is still less than that of the time series over the July 2008 to February 2009 period. As demonstrated in figure 6, the fitted seasonal patterns thus change dramatically between the March 2008 and March 2009 vintages. The pattern is similar to that exhibited above in figure 3. The seasonal components grow larger in magnitude after the additional year of data spanning the recession is included in the estimation procedure.

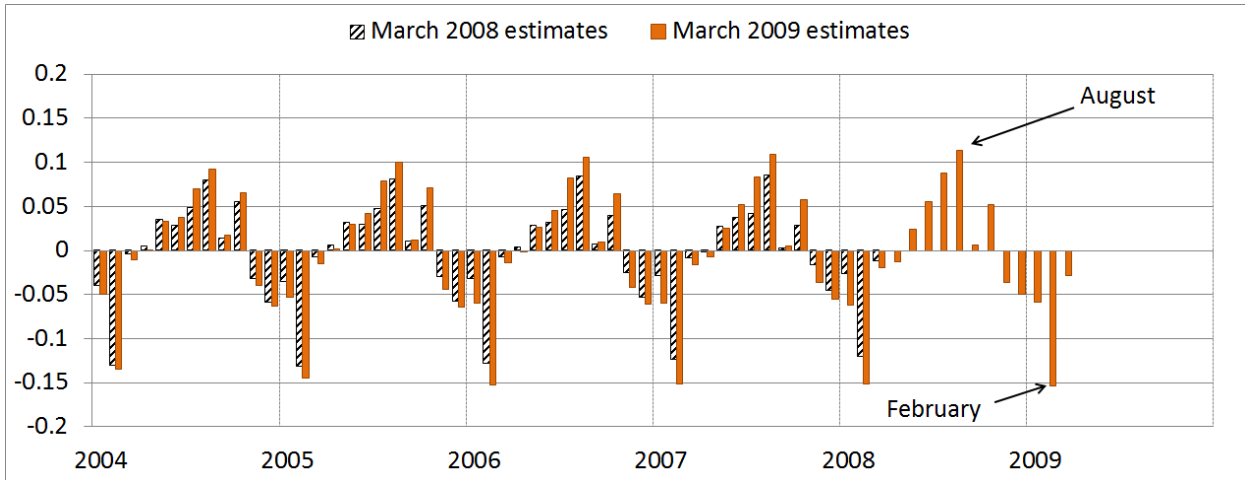


Figure 6: Vintage estimates of seasonal components to log crude oil imports. Trend component estimated using the 13 term Henderson trend filter; seasonal component estimated using the  $3 \times 9$  seasonal filter.

### 3.2.2 Multivariate Model

In contrast to the smoothing filters, the factor model (used to estimate the trend component under the multivariate approach) can easily accommodate abrupt changes in the level of the time series provided that the change is common (i.e., a large proportion of the time series experience a change in level at approximately the same time). This is because the factor model permits discontinuities in the slope of the fitted trend in order to capture the high frequency covariation in the cross section of time series.

The recession provides an example of a common, abrupt change in the level of a time series. Figure 7 below exhibits crude oil imports alongside other 5 digit level imports. (Refer to section 4 for details on these data.) All of the series experience a decline in late 2008 to a lesser or greater extent. Because the factor loadings are different for different time series, the factor model is sufficiently flexible to permit the decline in computer imports to be less severe than the decline in crude oil when modeling the covariation in the time series.

Figure 8 below depicts the trend component fitted under the multivariate approach, for the March 2008 and March 2009 vintages. (Refer to section 4 for specific details regarding the estimation of the trend.) There is no change between the March 2009 and March 2008 vintages for the period in which they overlap: This is due to the nature of the updating used in the estimation of the parametric model (see section 2.2 above). The panel dataset used in estimation of the factor model is all 5-digit level imports (additional details regarding estimation are described in the following section). The fitted trend component does a better job of tracking the decline in the time series over the August 2008 to February 2009 period

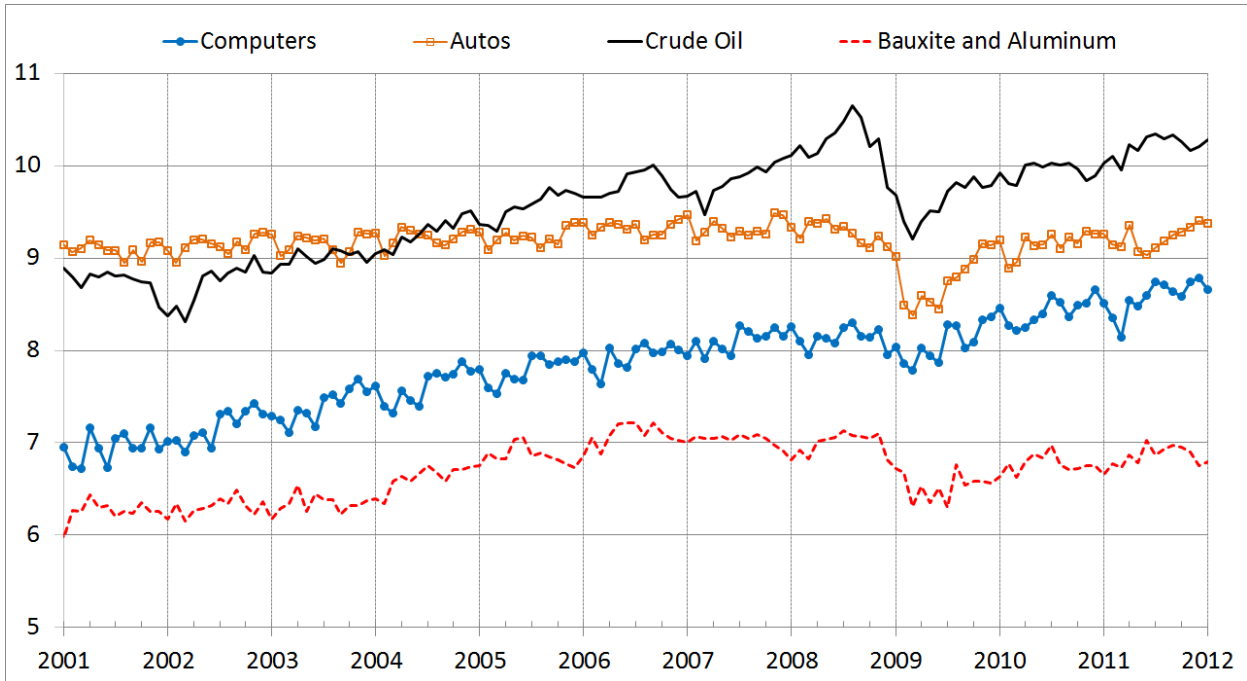


Figure 7: Covariation in selected log nominal imports.

than both the  $2 \times 12$  filter and the Henderson filter. (See figure 10 below for a direct comparison of the three fitted trends.) Consequently, the seasonal components depicted in figure 9 appear more stable, and in particular they do not change substantially between the March 2008 and March 2009 vintages.

The key difference between the smoothed trend (depicted in figures 1 and 5) and the multivariate trend is that the slope of the latter is discontinuous. This permits the multivariate trend to remain high through the summer of 2008, before plummeting between October and November 2008 along with the original time series. In contrast, both the Henderson and the  $2 \times 12$  filters are “pulled down” earlier by the precipitous decline. This point is demonstrated in figure 10.

### 3.3 Interventions

The weaknesses of the conventional X-11 procedures to large discontinuities in the slope of the time series are well known. The X-11 modules come with in-built methods to correct or attenuate the problems. In this subsection we briefly overview these interventions.

**Outliers.** An outlier intervention involves (essentially) omitting the period of the time series that is subject to irregular behavior. Because the recession is likely to generate pro-



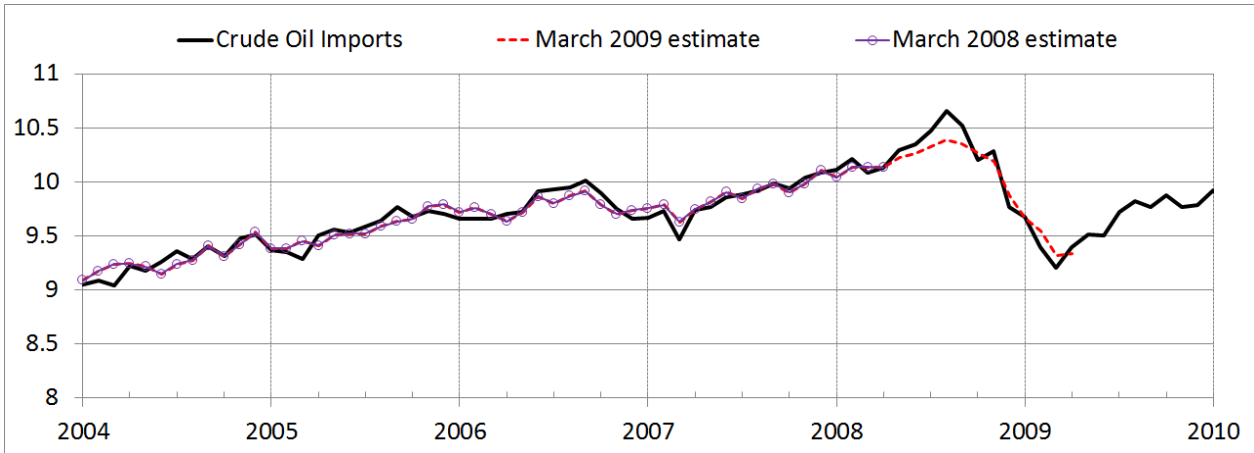


Figure 8: Log crude oil imports and factor model trend, March 2008 and March 2009 vintage estimates.

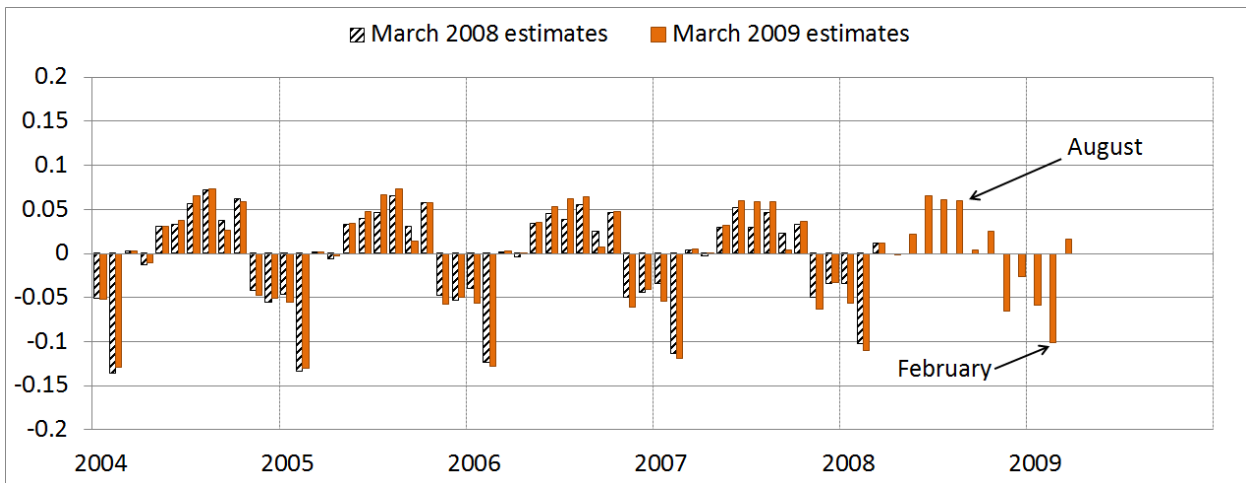


Figure 9: Multivariate filter-based seasonal components, log crude oil, March 2008 and March 2009 vintage estimates. Trend component estimated using a factor model.

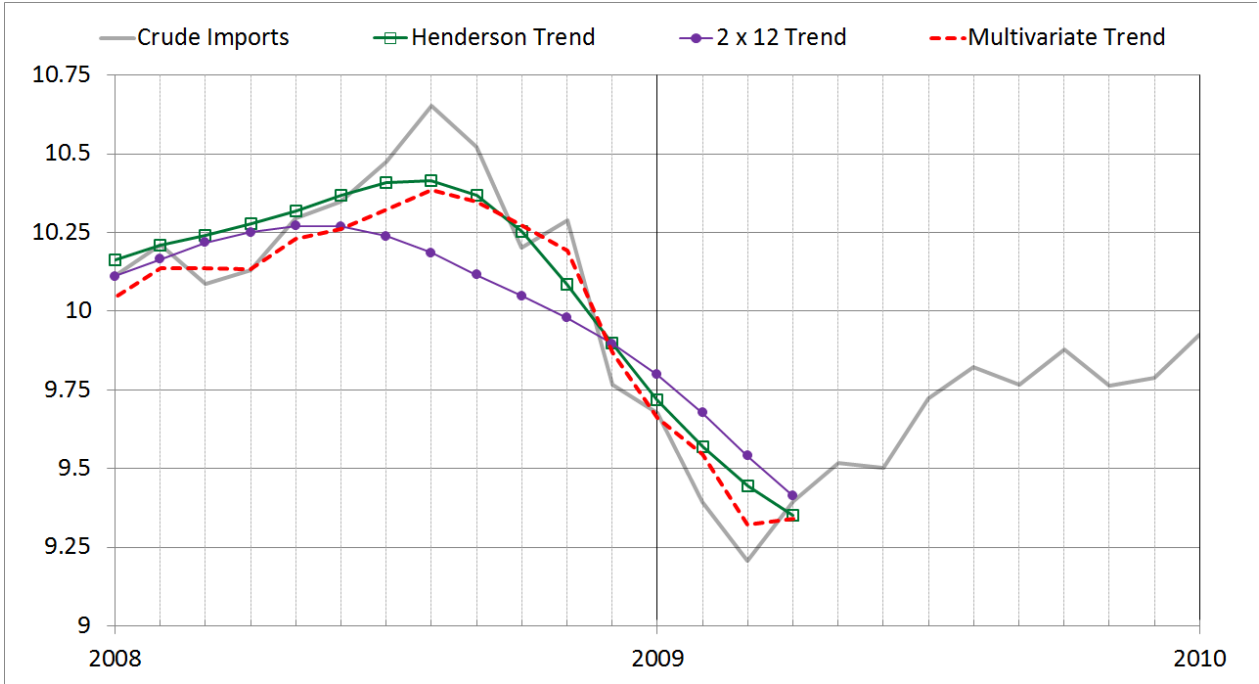


Figure 10: Estimates of trend component to log crude oil imports, March 2009 vintage estimates. Both the Henderson trend and  $2 \times 12$  trend are 13-month moving averages.

longed changes in the level of a time series, a sequence of outliers is likely to be required if using the treatment to deal with the effects of a recession. The regARIMA capability in X-12 permits outliers in the ARIMA modeling phase of the seasonal adjustment procedure, and so the ARIMA model is used to “fill in” parts of the series that are considered outliers. The X-12 package includes various statistical methods for identifying outliers according to statistical thresholds that can be set by the analyst.

**Trend Discontinuities.** As demonstrated in the previous subsection, the smoothness of the univariate trend is a disadvantage when tracking sharp inflection points. Interventions such as level shifts, temporary changes and ramps effectively introduce discontinuities into the fitted trend to better accommodate sharp discontinuities. A level shift introduces a single break in the trend between consecutive time periods; a temporary change introduces a level shift followed by a geometric decay back to the original level of the trend; and a ramp introduces a break between two non-consecutive time periods with a linear trend inserted between the break points. While there are statistical tests for determining the size and duration of a level shift or a temporary change, there are no tests for the size and duration of the ramp intervention.

The range of possible interventions is sufficiently broad to ensure that an analyst can

remove any distortions in the seasonal components. Indeed, there is substantial evidence to suggest that seasonally-adjusted data published by Federal statistical agencies are largely free of recession-related distortions (see., e.g., Macroeconomic Advisors, 2012; Kropf and Hudson, 2013; Evans and Tiller, 2013).

However, we raise two concerns regarding the use of interventions. The first is the lack of timeliness of the intervention. Applying the intervention requires that the analyst must select dates on when to begin and when to end the intervention. This can be particularly problematic when the seasonal adjustment needs to be performed while the recession is occurring, as it requires the analyst to select the peak and the trough dates in close to real time. If instead the analyst relies on formal statistical methods, several years may pass before the test selects the size and the dates of the break with any certainty. (Indeed, the asymptotic theory for determining the size and dates of a break require the time series on either side of the break to grow large (see, amongst others, Bai, 1996). Because the multivariate approach permits discontinuities in the fitted trend, it is not subject to these concerns.

The second concern is the transparency of the intervention. The nature of the intervention somewhat depends subjectivity of the analyst, meaning that it may be difficult for a third party to replicate the seasonal components published by a statistical agency. If statistical methods are used to select the type, duration and size of the intervention, then a third party must know the relevant thresholds and model selection criteria adopted in order to have a chance of replicating the series. If instead the subjective judgment of the analyst is used, then a third party will not be able to replicate the series.

Because the distortionary effects of the recession can be mitigated by use of these interventions, but only at after a substantial delay, we see the primary advantage of the multivariate approach as being the timely construction of robust seasonally-adjusted data. In comparing the two approaches in the next section we will estimate the seasonal model in near real time. In this sense we conduct concurrent seasonal adjustment exercise.

## 4 Empirical Evaluation

In this section we apply our new multivariate seasonal adjustment method to a panel data set of nominal goods imports. For comparative purposes we also apply the conventional filter-based procedures, using both the simple moving average and the Henderson filter. (Both are therefore univariate X-11 methods, although the Henderson filter is more likely to be used by statistical agencies.).

Many of the time series in the dataset exhibit either level shifts or tight downward inflections in late 2008 after the financial crisis sets in. The precipitous fall in trade is sufficiently pervasive to show up in aggregate trade numbers. Figure 11 below exhibits

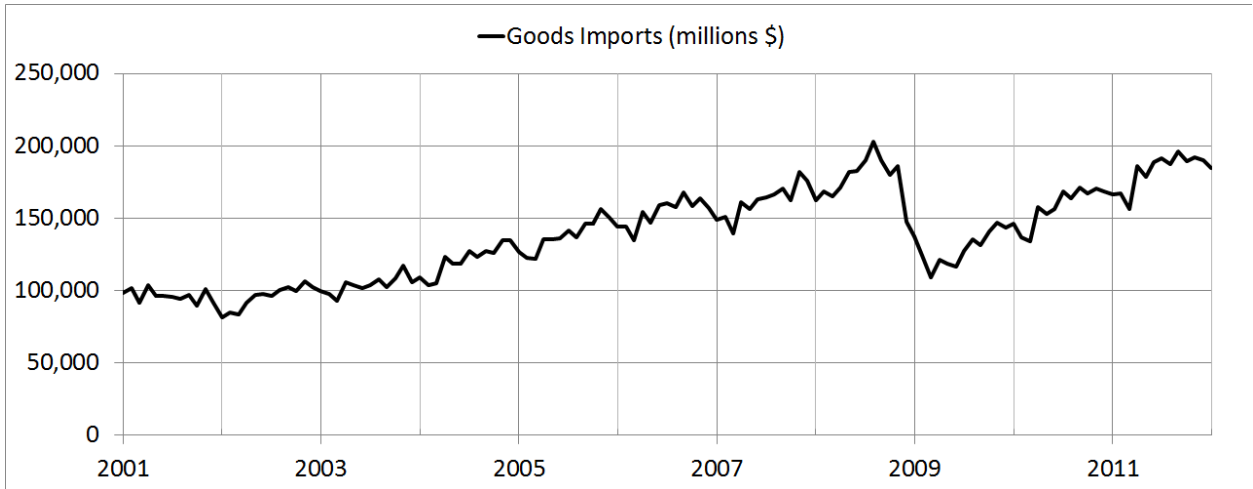


Figure 11: Monthly Nominal Goods Imports, 2001-2011.

monthly aggregate imports over the 2002 to 2012 period. Imports experience a large decline over the last half of 2008 and into early 2009. As discussed above, conventional smoothing filters cannot easily accommodate these abrupt changes in the trend of a time series, and hence we expect the multivariate approach to yield more stable seasonal adjustment factors over the broad recessionary period.

We propose two primary quantitative criteria for evaluating the two seasonal adjustment methods. Our primary evaluation criteria will be to test for distortions in the seasonal components of the time series. Our secondary criteria considers the revisions to seasonal components in a real-time updating exercise. Because one of our main evaluation criteria is revisions, we eliminate influences on revisions other than the effect of new data becoming available. In this regard:

- Asymmetric filters are used for estimating seasonal components at the endpoints of the sample. The use of symmetric filters requires the time series to be forecast and backcast from the endpoints of the sample. This is typically done by using ARIMA models (hence the X-11-ARIMA nomenclature). As time passes, the forecasted data are replaced with realized data, meaning that forecast error is a major component of revisions to seasonal components. By using asymmetric filters, we remove forecast error from the revisions.
- For the univariate approach, we use a centered  $2 \times 12$  month moving average as a first-pass estimate of the trend. This is a standard filter used in seasonal adjustment methods. For the Henderson univariate approach, a 13 term Henderson filter is used as a second-pass estimate of the trend before making a final estimate of the seasonal

components based on the Henderson-filtered time series. Both the  $2 \times 12$  and the Henderson filter use asymmetric filters at the end points of the sample.

- For both the univariate and multivariate approaches we use a  $3 \times 9$  moving average window for the seasonal filters.
- Seasonal components are re-estimated either at the end of every year (December), or at the end of every quarter (March, June, September or December), using the data available up to that point in calendar time.
- The factor model is estimated using a rolling window of 120 months (10 years) of time series. Rather than selecting a specific factor number, we use model averaging based on the Bai-Ng  $IC_3(k)$  criteria for each sub-sample. This particular criteria has the lowest penalty on model complexity of the three IC criteria, and we have greater concern of under-fitting the model rather than over-fitting the model.<sup>12</sup> The panel is first differenced and then standardized before the IC are computed. The maximum number of factors is set to the largest integer less than  $2 \times \min(\sqrt{T}, \sqrt{n})$ , which in this case is 22.<sup>13</sup>

In practice the span of the univariate filters (such as the  $2 \times 12$  trend, 13 term Henderson filter, and  $3 \times 9$  seasonal filter) can be selected based on relevant signal-to-noise ratios in the X-11 procedure (see, e.g., Australian Bureau of Statistics, 2005). In this empirical exercise we pre-set the span of the filters for two reasons. First, the  $3 \times 9$  seasonal filter spans eleven years of data and thus the relatively long span should help mitigate any distortionary effects of the recession. Thus, we expect the long span to aid the performance of the univariate approach. Second, preliminary analysis revealed that automated filter selection was markedly affected by the onset of the recession, with shorter seasonal filter spans being selected once the recession distorted the seasonal components. This worsened the performance of the univariate approach when evaluated by our criteria.

Updating the seasonal components at the end of every quarter corresponds to concurrent seasonal adjustment for data released at a quarterly frequency. As discussed at the end of section 3.3, we see the primary advantage of the multivariate approach being the timely

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<sup>12</sup>A selection criteria that is motivated within an information loss framework would be ideal in this application. Indeed, univariate seasonal adjustment models are often selected using AIC (Lytras, 2012), which minimizes Kullback-Leibler information loss. Unfortunately the author does not know of any formal criteria for factor model estimation, and so we rely on the popular Bai-Ng criteria.

<sup>13</sup>The maximum is set based on the conjecture that the maximum model dimensionality can grow at a rate slower than the rate of estimator convergence. Similar results have been proven for time series models (Shibata, 1980; Ing and Wei, 2005). As discussed in section 6.2, the rate of convergence for stationary panels is  $\min(\sqrt{n}, \sqrt{T})$ .

seasonal adjustment of time series. Concurrent seasonal adjustment contrasts against “projected factor” seasonal adjustment, in which the estimation of seasonal components is delayed until additional data are available, and the seasonal components from previous periods are used in the interim. The performance of a univariate filter approach is therefore going to depend on how soon after the close of the reference quarter the first estimate is made. (As additional time passes, the estimate is likely to be better, particularly if interventions are employed.) Rather than further complicate the analysis by considering the performance of the univariate and multivariate methods at different lags, we opt to use a concurrent seasonal adjustment approach. A projected-factor comparison is however worthy of future research.

## 4.1 Data and Seasonal Descriptive Statistics

Non-seasonally-adjusted data was obtained from the BEA, spanning January 1989 to December 2011. The data are obtained at the 5-digit level (the highest degree of disaggregation possible), although sparse time series were aggregated up to the 4-digit level for computational ease.<sup>14</sup> All data are logged prior to additive seasonal adjustment modeling. This corresponds to multiplicative seasonal adjustment of the original level of the series, although all evaluation criteria will be expressed in relation to the logged time series.

To convey the normal pattern of seasonality in the import series, Table 1 contains average seasonal components for the monthly (log) imports by quarter over the 1990 to 2007 period. We can see that total imports generally grow (on average) in quarters two and three, and fall in quarter one. Imports are generally flat in quarter four. This however masks some heterogeneity. Foods and consumer imports increase in quarter 4. In addition, the seasonality is more pronounced in foods and consumer goods.

Although the NBER recession begins in 2007, the fall in aggregate imports begins in late 2008. The exact timing of the fall in imports differs across the disaggregate imports. Tables 2 and 3 exhibit a frequency count of the quarter of the peak and the trough in the individual import series during the recession. The frequency counts are broken down by two digit level subgroups. Many of the Industrial Supplies and Capital imports peak over the summer and reach a trough in early 2009. The timing of the peak is slightly later for consumer goods, possibly due to holiday-driven demand for final goods.

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<sup>14</sup>If a particular time series exhibits zero entries in any time period, all items within the items’ 4-digit category are aggregated together to form a new time series. All 5-digit items within that particular 4-digit class are removed from the dataset.

Table 1: Seasonality in 5-digit-level Imports

Quarter	Foods	Industrial	Capital	Automotive	Consumer	Other	All
Q1	<b>-0.018</b>	-0.010	<b>-0.022</b>	-0.005	<b>-0.098</b>	<b>-0.016</b>	<b>-0.033</b>
Q2	<b>-0.007</b>	<b>0.020</b>	<b>0.011</b>	<b>0.032</b>	<b>-0.055</b>	<b>0.004</b>	-0.002
Q3	<b>-0.038</b>	-0.007	<b>-0.008</b>	<b>-0.064</b>	<b>0.061</b>	<b>-0.004</b>	0.002
Q4	<b>0.025</b>	<b>-0.019</b>	<b>0.010</b>	<b>0.024</b>	0.028	<b>0.013</b>	0.005

Table entries are average seasonal components implied by the log difference of seasonally-adjusted to non-seasonally-adjusted monthly imports published by the BEA. Averages are taken over January 1990 to December 2007. Bold font indicates statistical significance at the 5% level using (cross section heteroskedasticity) robust White (1980) standard errors. Two digit level sub group abbreviations are given in the note to Table 2.

Table 2: Frequency of local peaks in monthly import time series by quarter

Peak Date	Foods	Industrial	Capital	Automotive	Consumer	Other	All
Q1 2008 (or earlier)	4	8	4	3	4	0	23
Q2 2008	2	12	11	1	5	0	31
Q3 2008	3	26	10	0	7	4	50
Q4 2008 (or later)	8	5	4	0	14	1	32

Table entries are the number of peaks in the cross section of import series that occurred in the reference quarter. For example, a total of 23 import series had a peak in Q1 2008 or earlier. Two digit level sub group abbreviations are as follows: “Foods” refers to “Foods, feeds and beverages”, “Industrial” refers to “Industrial supplies & materials”, “Capital” refers to “Capital goods, except automotive”, “Automotive” refers to “Automotive vehicles, parts and engines”, “Consumer” refers to “Consumer goods, except automotive”, and “Other” refers to “Imports, not elsewhere specified.”

Table 3: Frequency of local troughs in monthly import time series by quarter

Trough Date	Foods	Industrial	Capital	Automotive	Consumer	Other	All
Q4 2008 (or earlier)	4	2	1	0	3	1	6
Q1 2009	7	22	12	2	19	3	65
Q2 2009	2	16	7	1	4	0	30
Q3 2009	2	8	6	1	2	1	20
Q4 2009 (or later)	2	3	3	0	2	0	10

Table entries are the number of troughs in the cross section of import series that occurred in the reference quarter. For example, a total of 65 import series had a trough in Q1 2009.

## 4.2 Statistical Tests for Distortion of Concurrent Seasonal Components

In this subsection we test for distortion in the fitted seasonal components that is brought about by the 2007-2009 recession. A simple approach would be to see if the fitted seasonals change around the 2008 period. However the nonparametric seasonal filter used in both the univariate and the multivariate approach is sufficiently flexible to permit fitted seasonals to change over time, and thus changes in the fitted seasonal components of an individual time series may be reflective of genuine changes in seasonal patterns in the time series that coincide with the 2007-2009 period, rather than of spurious changes in seasonals brought about the deceleration in economic activity. That said, it seems unlikely that in a large enough cross section of times series, we would not observe a genuine, *common* change in seasonal patterns over the recession. A common change is therefore more indicative of distorted seasonal components.

What would a common distortionary effect look like? As discussed in the introduction, the timing of the recession is likely to make the seasonal components small in quarters four and one, and larger in quarters two and three, if the trend component “under-fits” the decline (that is, rate of change in the fitted trend is less than that of the time series). Note that this holds regardless of the seasonal patterns of the specific time series in question. Thus although consumer goods tend to grow in quarter 4, while other categories are flat or decline, this does not matter when looking for distortionary effects once we condition on the level of the seasonal component.<sup>15</sup>

To test for a common change in the fitted seasonal components, we estimate a regression

<sup>15</sup>Thus, if the fitted trend understates the fall in economic activity, we expect to see the recession distorting seasonal factors by making them larger in magnitude.



of the form

$$\hat{s}_{i,j,s} = \alpha_{i,j} + \beta_s + u_{i,j,s}, \quad j \in J, \quad J \in \{Q_1, Q_2, Q_3, Q_4\}$$

Here  $s$  indexes the year, and  $j = 1, \dots, 12$  indexes the month, so that  $\hat{s}_{i,j,s}$  denotes the estimated seasonal component for month  $j$  of year  $s$  for the  $i$ th cross section. The sets  $Q_1, Q_2$ , etc., correspond to the months in each quarter of the year, i.e.,

$$Q_k = \{j : 3 \times (k - 1) + 1 \leq j \leq 3 \times k\}, \quad k = 1, 2, 3, 4.$$

Thus we pool across cross sections and across months within each quarter when testing a pervasive change in seasonal components. The (cross section) fixed effect  $\alpha_{i,j}$  captures the time-series average of the month  $j$  seasonal component of the  $i$ th cross section over the time period considered. For example, if the seasonal component for (log) crude oil is negative on average in February, then the fixed effect for February crude oil will be negative. The common time effects  $\beta_s$  then pick up any common trend in the seasonal components (across all cross sections) within each quarter after conditioning on the fixed effects. For example, if the seasonal component for the first quarter are lower at the end of the sample than at the beginning (on average across cross sections), then this will be reflected in the sequence of fitted  $\beta_s$  going from positive to negative over the sample. We interpret statistical significance in the fitted  $\beta_s$  as distortion in the seasonal components.

#### 4.2.1 Univariate Seasonal Adjustment

Table 4 below exhibits the point estimates of the dummy variables  $\beta_s$  for the univariate seasonal components, as estimated using data up to and including December 2009. Bold font indicates the estimate is statistically different from zero at the 95% level. 115 of the 120 points estimates are statistically different from zero, indicating that the components exhibit a clear pattern across the 136 export series over the 2000 to 2009 period. The pattern is clear: seasonals in quarters three and four grow larger over the sample period, while seasonals in quarters one and two grow smaller. This is consistent with a pervasive decline in all the time series that ends in the first quarter of 2009, that is not accommodated in the fitted trends.

Tables 5 through 7 below exhibit the point estimates of the dummy variables for the March 2009, June 2009, and September 2009 vintages. In each case, distortion in the seasonal components is evident.

#### 4.2.2 Univariate Henderson Filter Seasonal Adjustment

Table 8 exhibits the point estimates of the dummy variables  $\beta_s$  for the December 2009 vintage of univariate seasonal components based on the second-stage Henderson trend filter.

Table 4: Test for Distortion in Univariate Seasonals, 2 x 12 trend filter, December 2009 vintage

Depend. Var.	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Q1 Seasonals	<b>0.51</b>	<b>0.48</b>	<b>0.47</b>	<b>0.40</b>	<b>0.22</b>	<b>-0.20</b>	<b>-0.40</b>	<b>-0.47</b>	<b>-0.42</b>	<b>-0.58</b>
Q2 Seasonals	0.22	<b>0.29</b>	<b>0.36</b>	<b>0.33</b>	<b>0.16</b>	0.05	-0.11	<b>-0.20</b>	<b>-0.42</b>	<b>-0.69</b>
Q3 Seasonals	-0.34	<b>-0.41</b>	<b>-0.47</b>	<b>-0.32</b>	<b>-0.12</b>	<b>0.18</b>	<b>0.27</b>	<b>0.29</b>	<b>0.32</b>	<b>0.61</b>
Q4 Seasonals	<b>-0.39</b>	<b>-0.35</b>	<b>-0.36</b>	<b>-0.41</b>	<b>-0.26</b>	-0.03	<b>0.24</b>	<b>0.38</b>	<b>0.52</b>	<b>0.65</b>

Estimates from a regression of seasonal components on year dummy variables. Point estimates are multiplied by 100. Bold face font denotes statistical significance at the 95% level. Standard errors are clustered within cross sections. Trend is estimated using a  $2 \times 12$  trend filter, seasonals estimated using a  $3 \times 9$  filter.

Table 5: Test for Distortion in Univariate Seasonals, 2 x 12 trend filter, March 2009 vintage

Depend. Var.	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Q1 Seasonals	-0.19	-0.10	-0.03	0.03	0.04	<b>0.10</b>	0.11	0.08	0.03	-0.07
Q2 Seasonals	-0.12	-0.24	<b>-0.31</b>	<b>-0.38</b>	<b>-0.20</b>	0.05	<b>0.30</b>	<b>0.28</b>	<b>0.27</b>	<b>0.34</b>
Q3 Seasonals	-0.16	-0.12	-0.08	-0.11	<b>-0.11</b>	-0.04	0.04	0.10	0.17	<b>0.32</b>
Q4 Seasonals	<b>0.47</b>	<b>0.46</b>	<b>0.42</b>	<b>0.46</b>	<b>0.27</b>	<b>-0.11</b>	<b>-0.45</b>	<b>-0.47</b>	<b>-0.47</b>	<b>-0.59</b>

Estimates from a regression of seasonal components on year dummy variables. Point estimates are multiplied by 100. Bold face font denotes statistical significance at the 95% level. Standard errors are clustered within cross sections. Trend is estimated using a  $2 \times 12$  trend filter, seasonals estimated using a  $3 \times 9$  filter.

Table 6: Test for Distortion in Univariate Seasonals, 2 x 12 trend filter, June 2009 vintage

Depend. Var.	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Q1 Seasonals	-0.18	<b>-0.30</b>	<b>-0.36</b>	<b>-0.44</b>	<b>-0.22</b>	0.03	<b>0.34</b>	<b>0.32</b>	<b>0.35</b>	<b>0.45</b>
Q2 Seasonals	-0.21	-0.17	-0.12	<b>-0.16</b>	<b>-0.14</b>	-0.06	0.08	0.13	<b>0.25</b>	<b>0.41</b>
Q3 Seasonals	<b>0.53</b>	<b>0.51</b>	<b>0.48</b>	<b>0.52</b>	<b>0.31</b>	<b>-0.13</b>	<b>-0.49</b>	<b>-0.54</b>	<b>-0.53</b>	<b>-0.67</b>
Q4 Seasonals	-0.13	-0.04	0.01	0.08	0.05	<b>0.15</b>	0.07	0.08	-0.07	-0.20

Estimates from a regression of seasonal components on year dummy variables. Point estimates are multiplied by 100. Bold face font denotes statistical significance at the 95% level. Standard errors are clustered within cross sections. Trend is estimated using a  $2 \times 12$  trend filter, seasonals estimated using a  $3 \times 9$  filter.

Table 7: Test for Distortion in Univariate Seasonals, 2 x 12 trend filter, September 2009 vintage

Depend. Var.	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Q1 Seasonals	-0.26	-0.23	-0.18	<b>-0.30</b>	<b>-0.23</b>	-0.11	<b>0.17</b>	<b>0.23</b>	<b>0.38</b>	<b>0.52</b>
Q2 Seasonals	<b>0.57</b>	<b>0.54</b>	<b>0.51</b>	<b>0.47</b>	<b>0.26</b>	<b>-0.18</b>	<b>-0.45</b>	<b>-0.52</b>	<b>-0.50</b>	<b>-0.69</b>
Q3 Seasonals	0.22	<b>0.30</b>	<b>0.35</b>	<b>0.34</b>	<b>0.18</b>	0.08	-0.12	<b>-0.19</b>	<b>-0.43</b>	<b>-0.72</b>
Q4 Seasonals	<b>-0.53</b>	<b>-0.60</b>	<b>-0.68</b>	<b>-0.51</b>	<b>-0.21</b>	<b>0.21</b>	<b>0.40</b>	<b>0.48</b>	<b>0.55</b>	<b>0.89</b>

Estimates from a regression of seasonal components on year dummy variables. Point estimates are multiplied by 100. Bold face font denotes statistical significance at the 95% level. Standard errors are clustered within cross sections. Trend is estimated using a  $2 \times 12$  trend filter, seasonals estimated using a  $3 \times 9$  filter..

109 of the 120 points estimates are statistically different from zero, indicating distortion in the seasonal components. This is a small improvement on the univariate with  $12 \times 2$  filter, exhibited in table 4. The pattern remains the same as before: seasonals in quarters three and four grow larger over the sample period, while seasonals in quarters one and two grow smaller.

Table 8: Test for Distortion in Univariate Seasonals, Henderson filter, December 2009 vintage

Depend. Var.	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Q1 Seasonals	<b>0.39</b>	<b>0.32</b>	<b>0.27</b>	<b>0.25</b>	<b>0.16</b>	<b>-0.11</b>	<b>-0.30</b>	<b>-0.34</b>	<b>-0.33</b>	<b>-0.31</b>
Q2 Seasonals	0.09	0.14	<b>0.18</b>	<b>0.13</b>	0.03	0.00	-0.04	-0.07	-0.17	<b>-0.31</b>
Q3 Seasonals	-0.21	<b>-0.24</b>	<b>-0.24</b>	<b>-0.19</b>	<b>-0.09</b>	0.08	<b>0.18</b>	<b>0.20</b>	<b>0.22</b>	<b>0.30</b>
Q4 Seasonals	-0.27	<b>-0.23</b>	<b>-0.21</b>	<b>-0.19</b>	<b>-0.11</b>	0.03	<b>0.17</b>	<b>0.22</b>	<b>0.28</b>	<b>0.32</b>

Estimates from a regression of seasonal components on year dummy variables. Point estimates are multiplied by 100. Bold face font denotes statistical significance at the 95% level. Standard errors are clustered within cross sections. Second stage trend estimated using Henderson filter, with asymmetric filter at the endpoints. Seasonals estimated using a  $3 \times 9$  filter.

Tables 9 through 11 below exhibit the point estimates of the dummy variables for the March 2009, June 2009, and September 2009 vintages. In each case, distortion in the seasonal components become more evident as the vintage progresses.

Table 9: Test for Distortion in Univariate Seasonals, Henderson filter, March 2009 vintage

Depend. Var.	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Q1 Seasonals	-0.20	-0.12	-0.06	0.00	0.00	0.06	0.09	0.09	0.09	0.04
Q2 Seasonals	-0.12	-0.14	-0.16	<b>-0.18</b>	<b>-0.11</b>	0.02	<b>0.15</b>	<b>0.18</b>	0.19	0.19
Q3 Seasonals	-0.10	-0.07	-0.05	-0.08	<b>-0.08</b>	0.00	0.06	0.10	0.10	0.13
Q4 Seasonals	<b>0.42</b>	<b>0.34</b>	<b>0.27</b>	<b>0.26</b>	<b>0.19</b>	-0.08	<b>-0.30</b>	<b>-0.37</b>	<b>-0.37</b>	<b>-0.36</b>

Estimates from a regression of seasonal components on year dummy variables. Point estimates are multiplied by 100. Bold face font denotes statistical significance at the 95% level. Standard errors are clustered within cross sections. Second stage trend estimated using Henderson filter, with asymmetric filter at the endpoints. Seasonals estimated using a  $3 \times 9$  filter.

Table 10: Test for Distortion in Univariate Seasonals, Henderson Filter, June 2009 vintage

Depend. Var.	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Q1 Seasonals	-0.18	-0.19	<b>-0.20</b>	<b>-0.19</b>	<b>-0.11</b>	0.03	<b>0.16</b>	<b>0.20</b>	<b>0.23</b>	<b>0.25</b>
Q2 Seasonals	-0.13	-0.09	-0.06	-0.07	<b>-0.07</b>	0.00	0.06	0.10	0.11	0.15
Q3 Seasonals	<b>0.45</b>	<b>0.36</b>	<b>0.29</b>	<b>0.29</b>	<b>0.20</b>	<b>-0.09</b>	<b>-0.32</b>	<b>-0.39</b>	<b>-0.40</b>	<b>-0.38</b>
Q4 Seasonals	-0.14	-0.08	-0.03	-0.02	-0.01	0.06	0.10	0.09	0.05	-0.02

Estimates from a regression of seasonal components on year dummy variables. Point estimates are multiplied by 100. Bold face font denotes statistical significance at the 95% level. Standard errors are clustered within cross sections. Second stage trend estimated using Henderson filter, with asymmetric filter at the endpoints. Seasonals estimated using a  $3 \times 9$  filter.

Table 11: Test for Distortion in Univariate Seasonals, Henderson filter, September 2009 vintage

Depend. Var.	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Q1 Seasonals	-0.20	-0.15	-0.10	-0.11	<b>-0.09</b>	0.00	0.09	0.15	0.19	0.23
Q2 Seasonals	<b>0.45</b>	<b>0.36</b>	<b>0.29</b>	<b>0.26</b>	<b>0.18</b>	<b>-0.11</b>	<b>-0.32</b>	<b>-0.38</b>	<b>-0.37</b>	<b>-0.36</b>
Q3 Seasonals	0.11	0.16	<b>0.19</b>	<b>0.14</b>	0.05	0.01	-0.04	-0.09	-0.19	<b>-0.34</b>
Q4 Seasonals	<b>-0.35</b>	<b>-0.37</b>	<b>-0.38</b>	<b>-0.30</b>	<b>-0.13</b>	0.10	<b>0.27</b>	<b>0.32</b>	<b>0.37</b>	<b>0.48</b>

Estimates from a regression of seasonal components on year dummy variables. Point estimates are multiplied by 100. Bold face font denotes statistical significance at the 95% level. Standard errors are clustered within cross sections. Second stage trend estimated using Henderson filter, with asymmetric filter at the endpoints. Seasonals estimated using a  $3 \times 9$  filter.

### 4.2.3 Multivariate Seasonal Adjustment

In contrast to the univariate seasonal components exhibited in tables 4 and 8, there is far less evidence of distortion in the December 2009 vintage of multivariate seasonals. As shown in table 12, only 25% of the fitted seasonals are statistically significant.

Table 12: Regression Test for Distortion in Multivariate Seasonals, December 2009 vintage

Depend. Var.	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Q1 Seasonals	-0.04	0.01	0.06	0.09	<b>0.10</b>	-0.03	-0.09	-0.06	-0.04	-0.01
Q2 Seasonals	<b>-0.32</b>	<b>-0.24</b>	-0.15	-0.07	-0.01	<b>0.11</b>	<b>0.17</b>	<b>0.19</b>	0.18	0.15
Q3 Seasonals	-0.04	-0.06	-0.10	-0.08	-0.04	0.04	0.09	0.07	0.06	0.06
Q4 Seasonals	<b>0.39</b>	<b>0.29</b>	<b>0.19</b>	0.06	-0.05	<b>-0.13</b>	<b>-0.16</b>	<b>-0.20</b>	-0.20	-0.21

Estimates from a regression of seasonal components on year dummy variables. Point estimates are multiplied by 100. Bold face font denotes statistical significance at the 95% level. Standard errors are clustered within cross sections. Seasonals estimated using a  $3 \times 9$  filter.

Tables 13 through 15 below exhibit the point estimates of the dummy variables for the March 2009, June 2009, and September 2009 vintages. In each case, distortion in the seasonal components is not that evident.

Table 13: Regression Test for Distortion in Multivariate Seasonals, March 2009 vintage

Depend. Var.	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Q1 Seasonals	-0.19	-0.16	-0.11	-0.06	-0.04	0.04	0.11	0.13	0.15	0.12
Q2 Seasonals	0.02	0.01	-0.01	-0.05	-0.04	0.01	0.06	0.04	0.00	-0.04
Q3 Seasonals	0.30	0.21	0.11	0.03	-0.04	-0.07	-0.10	-0.14	-0.16	-0.14
Q4 Seasonals	-0.13	-0.06	0.00	0.08	<b>0.11</b>	0.02	-0.06	-0.03	0.01	0.05

Estimates from a regression of seasonal components on year dummy variables. Point estimates are multiplied by 100. Bold face font denotes statistical significance at the 95% level. Standard errors are clustered within cross sections. Seasonals estimated using a  $3 \times 9$  filter.

### 4.3 Revisions to Seasonal Components.

Seasonal components at the endpoints of a time series are either based on asymmetric moving averages (as in conventional univariate X-11) or they are based on symmetric filters using

Table 14: Regression Test for Distortion in Multivariate Seasonals, June 2009 vintage

Depend. Var.	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Q1 Seasonals	0.08	0.06	0.02	-0.04	<b>-0.06</b>	-0.02	0.03	0.02	-0.02	-0.07
Q2 Seasonals	0.33	0.23	0.13	0.03	-0.05	<b>-0.10</b>	-0.12	-0.15	-0.16	-0.15
Q3 Seasonals	-0.08	-0.03	0.03	0.09	<b>0.11</b>	-0.01	-0.08	-0.06	-0.01	0.04
Q4 Seasonals	<b>-0.33</b>	<b>-0.26</b>	<b>-0.17</b>	-0.08	-0.01	<b>0.12</b>	<b>0.18</b>	<b>0.19</b>	0.18	0.18

Estimates from a regression of seasonal components on year dummy variables. Point estimates are multiplied by 100. Bold face font denotes statistical significance at the 95% level. Standard errors are clustered within cross sections. Seasonals estimated using a  $3 \times 9$  filter.

Table 15: Regression Test for Distortion in Multivariate Seasonals, September 2009 vintage

Depend. Var.	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Q1 Seasonals	0.28	0.20	0.10	0.00	<b>-0.07</b>	<b>-0.09</b>	-0.09	-0.10	-0.12	-0.11
Q2 Seasonals	-0.08	-0.02	0.03	0.09	<b>0.10</b>	-0.02	-0.09	-0.05	0.00	0.03
Q3 Seasonals	<b>-0.28</b>	<b>-0.21</b>	-0.13	-0.06	0.00	<b>0.10</b>	<b>0.15</b>	0.16	0.15	0.13
Q4 Seasonals	0.08	0.04	-0.01	-0.03	-0.03	0.01	0.03	0.00	-0.03	-0.05

Estimates from a regression of seasonal components on year dummy variables. Point estimates are multiplied by 100. Bold face font denotes statistical significance at the 95% level. Standard errors are clustered within cross sections. Seasonals estimated using a  $3 \times 9$  filter.

forecasted data (as in X-11-ARIMA and later versions of X-11). As more data become available, those seasonal components (that were previously at the end of the time series) are re-estimated as the new additional data are used in the estimation procedure. Revisions to seasonal components may also come about because of revisions in the underlying data itself.

Revisions to seasonal components are likely to be of concern to data compilers and users relying on timely data. Fixler and Grimm (2002) and Fixler Grimm and Lee (2003) show that revisions to seasonal components make a significant contribution to revisions in the US national accounts. For disaggregate nominal imports the revisions to seasonal components are on average almost as large as the revisions to the seasonally-adjusted data (Fixler Grimm and Lee, 2003). We therefore consider how the univariate and multivariate approaches perform when evaluated according to revisions. Under this metric, a seasonal adjustment procedure will be viewed positively if the seasonal components exhibit relatively small revisions.

We begin with a ten year dataset spanning January 1989 to December 1998. We update the sample annually by adding in an additional 12 months of data, and re-estimate the seasonal components. For the multivariate approach, we use the “rolling window” method outlined in subsection 2.2 for estimation of the parametric model, using the past 120 months of data to fit the factor model. We can then compute revisions to these seasonal components. To fix ideas, let  $\hat{s}_{i,j,s}(p)$  denote the seasonal component for the  $j$ th month in year  $s$  computed in year  $s + p$  for the  $i$ th cross section in the panel. We define the  $s_2$  to  $s_1$  annual revision to the seasonal component for month  $j$  in year  $s$  as follows.

$$r_{i,j,s}(p_2, p_1) = \hat{s}_{i,j,s}(p_2) - \hat{s}_{i,j,s}(p_1), \quad p_2 > p_1 \geq 0.$$

We consider first-to-second annual revisions (i.e.  $r_{i,j,s}(1, 0)$ ), second-to-third annual revisions (i.e.  $r_{i,j,s}(2, 1)$ ), third-to-fourth annual revisions (i.e.  $r_{i,j,s}(3, 2)$ ), and fourth-to-fifth annual revisions (i.e.  $r_{i,j,s}(3, 2)$ ). We consider two measures of the magnitude of revisions: mean absolute revisions (MAR) and root mean square revisions (RMSR).

Table 16 exhibits the MARs and RMSRs for the nominal imports dataset over the 1998-2011 time period. It is apparent that the multivariate approach leads to smaller revisions. The MARs of the multivariate approach are between 9% and 20% smaller than that of the univariate approach, while the RMSRs are between 12% and 21% smaller.

## 5 Concluding Remarks

In this paper we propose a new multivariate approach to seasonal adjustment. The approach bears all the main hallmarks of the univariate filter-based methods (such as X-11, X-11-ARIMA, X-12-ARIMA, and X-13-ARIMA-SEATS), but differs in how the stochastic



Table 16: Annual Revisions to Seasonal Components, Imports

Univariate ( $2 \times 12$ filter)					
	1st to 2nd	2nd to 3rd	3rd to 4th	4th to 5th	5th to 6th
RMSR	1.46	1.261	1.111	0.758	0.594
MAR	1.16	1.007	0.89	0.615	0.519
Univariate (13 term Henderson filter)					
RMSR	1.279	1.111	0.980	0.707	0.593
MAR	1.019	0.887	0.786	0.574	0.51
Multivariate					
RMSR	0.974	0.853	0.761	0.531	0.497
MAR	0.791	0.696	0.624	0.444	0.454

Root mean square revision (RMSR) and mean absolute revision (MAR) seasonal components for 5-digit level goods imports, 1998-2011. Seasonal components are re-estimated every December.

Averages taken across 136 import series. Seasonals estimated using a  $3 \times 9$  filter.

trend in the time series is conceptualized. Whereas the moving average filters used in the univariate methods yield smooth estimates of the trend component, under the multivariate approach the trend component is modelled using a parametric model of covariation. This permits the trend to accommodate large, common discontinuities in the cross section of time series under consideration, such as those brought about by severe recessions. Because the multivariate approach accommodates these sharp discontinuities in the fitted trends, the seasonal components are less prone to distortion resulting from severe business cycle fluctuations than univariate filter-based seasonal adjustment methods.

Our limited empirical analysis using nominal import data is encouraging. The multivariate approach appears to deal with the effects of the recession much better than the univariate filter-based methods. However, further empirical analysis is required to analyze the performance of the procedure in different contexts.

## 6 Appendix

### 6.1 Overview of X-11 Seasonal Adjustment

In this section we outline the conventional seasonal adjustment model. X-11 and its variants require decomposing the time series into a trend, seasonal and irregular component, using a sequence of pre-specified moving average filters.

- The trend component models low frequency variation in the series. Trend filters are weighted averages of consecutive months or quarters.

- An  $n \times m$  moving average is an  $m$ -term simple average taken over  $n$  consecutive periods, with equal weights attributed to each period. An example of a  $2 \times 12$  trend filter for March 2007 (2007:3) is as follows:

$$\begin{array}{cccccccccccc} 2006:10 & + & 2006:11 & + & \dots & + & 2007:3 & + & \dots & + & 2007:8 & + & 2007:9 & + \\ 2006:9 & + & 2006:10 & + & \dots & + & 2007:3 & + & \dots & + & 2007:9 & + & 2007:10 & \end{array}$$

This corresponds to the following weights in the 13 term weighted average:

2006:9	2006:10	2006:11	2006:12	2007:1	2007:2	2007:3
0.042	0.083	0.083	0.083	0.083	0.083	0.083
2007:4	2007:5	2007:6	2007:8	2007:9	2007:10	
0.083	0.083	0.083	0.083	0.083	0.042	

- The Henderson filter uses a non-equal weighting scheme. For a 13-term Henderson filter, the weights are as follows:

2006:9	2006:10	2006:11	2006:12	2007:1	2007:2	2007:3
-0.019	-0.028	0	0.066	0.147	0.214	0.240
	2007:4	2007:5	2007:6	2007:8	2007:9	2007:10
	0.214	0.147	0.066	0	-0.028	-0.019

Asymmetric Henderson filters are possible for the endpoints of the time series. See chapter 5 of Australian Bureau of Statistics (2005).

- The de-trended series is then used to estimate the seasonal components by taking a moving average over successive months (or quarters or weeks). Seasonal filters are computed using values from the same month or quarter, for example, an estimate for January would come from a weighted average of the surrounding Januaries. The seasonal filters available in X-12-ARIMA consist of seasonal moving averages of consecutive values within a given month or quarter. An  $n \times m$  moving average is an  $m$ -term simple average taken over  $n$  consecutive sequential spans. An example of a  $3 \times 5$  filter for January 2007 (2007:1) is:

$$\begin{array}{cccccc} 2005:1 & + & 2006:1 & + & 2007:1 & + & 2008:1 & + & 2009:1 & + \\ 2004:1 & + & 2005:1 & + & 2006:1 & + & 2007:1 & + & 2008:1 & + \\ 2003:1 & + & 2004:1 & + & 2005:1 & + & 2006:1 & + & 2007:1 & \end{array}$$

This corresponds to the following weights in the seven term weighted average:

2003:1	2004:1	2005:1	2006:1	2007:1	2008:1	2009:1
0.067	0.133	0.2	0.2	0.2	0.133	0.067

- Having obtained the seasonal components for each month or quarter, the seasonal components are refined by recursively de-trending the series for each new set of factors. That is, the original series  $x_t$  has the factors subtracted from it and the trend re-estimated. The series  $x_t$  is de-trended and the seasonal components re-estimated. The final trend may be based on the Henderson filter, before making the last estimate of the seasonal components.

## 6.2 Conventional factor model estimation

Let  $\mathbf{X} = (x_{i,t})$  denote a  $T \times n$  matrix of the panel data. In matrix notation, the factor model is

$$\mathbf{X} = \mathbf{F}\mathbf{\Lambda}' + \mathbf{u} \tag{4}$$

where  $\mathbf{F}$  is a  $T \times m$  matrix of stochastic factors,  $\mathbf{\Lambda}$  a  $n \times m$  matrix of factor loadings, and  $\mathbf{u}$  is a  $T \times n$  matrix of idiosyncratic components.  $\mathbf{F}\mathbf{\Lambda}'$  is a matrix of the common components. Key identification assumptions can be found in Bai (2003, 2004). However, provided that  $\mathbf{F}$ ,  $\mathbf{\Lambda}$  and  $\mathbf{u}$ , are statistically independent, a key assumption is that the idiosyncratic component is weakly dependent in the following sense (Amengual and Watson, 2007)

**Assumption A** (*Weak dependence in idiosyncratic components*)

$$\text{tr} \left( \frac{1}{nT} \mathbf{u}\mathbf{u}' \right)^j = O_p \left( \frac{1}{\min(n,T)} \right)$$

for some  $j \geq 2$ .

Any strong-form dependence in the panel is therefore generated by the factor structure, as implied by the following

**Assumption B** (*Strong dependence in common components*)

$$\frac{1}{n} \mathbf{\Lambda}' \mathbf{\Lambda} = O_p(1), \quad \frac{1}{T^2} \mathbf{F}' \mathbf{F} = O_p(1)$$

Assumption B ensures that the common factors are non-degenerate and that each factor has a non-trivial contribution to the variance of the time series. The common factors and loadings thereby generate strong-form dependence between the cross sections and time series in the panel. Additional assumptions requiring that the higher order moments of the stochastic variables are bounded are also required for consistent estimation. We refer the reader to Bai and Ng (2002) and Bai (2003, 2004) for additional details.

The common factors are modelled as  $I(1)$  time series, implying that the variance of the factors increases with  $T$  and hence we require that  $\mathbf{F}'\mathbf{F}$  be scaled by  $\frac{1}{T^2}$  to ensure it is bounded in probability under assumption Assumption B. We need not restrict ourselves to the non-stationary case however: The factors can be  $I(0)$  without changing the methods applied herein. In this case we would have  $\frac{1}{T} \mathbf{F}'\mathbf{F} = O_p(1)$ .

### 6.3 Principal components

Under these conditions, the common factors and factor loadings can be identified using the eigen-decomposition of the covariance matrix. Specifically, the first  $m$  orthonormal eigenvectors of  $\frac{1}{nT}\mathbf{X}\mathbf{X}'$  (associated with the largest  $m$  eigenvalues) multiplied by  $\sqrt{T}$  provide estimates of  $\mathbf{F}$ . This is the ‘‘principal components’’ estimator of the model, and the estimated factors solve the following problem.

$$\hat{\mathbf{F}} = \arg \min_{\mathbf{F}_0} \text{tr} \left( \frac{1}{nT} (\mathbf{X} - \frac{1}{T}\mathbf{F}_0\mathbf{F}_0'\mathbf{X}) (\mathbf{X} - \frac{1}{T}\mathbf{F}_0\mathbf{F}_0'\mathbf{X})' \right) \text{ s.t. } \mathbf{F}_0'\mathbf{F}_0 = \mathbf{I}_m$$

The estimated common factors  $\hat{\mathbf{F}}$  are consistent estimates of  $\mathbf{F}$  the sense that

$$T^{-1} \left\| \hat{\mathbf{F}} - \mathbf{F}\mathbf{H} \right\|^2 = O_p \left( \frac{1}{\min(\sqrt{n}, T)} \right)$$

where  $\mathbf{H}$  is a positive-definite matrix (Bai, 2004). Note that consistent estimation of the factor space requires that both  $n \rightarrow \infty$  and  $T \rightarrow \infty$ , but without restriction on the relative rates of expansion. If the common factors are  $I(0)$ , the convergence rates slows to  $\min(\sqrt{n}, \sqrt{T})$  (Bai, 2003). In either the  $I(1)$  or the  $I(0)$  factor case, the factor loadings can be estimated as  $\hat{\mathbf{\Lambda}} = T^{-1}\mathbf{X}'\hat{\mathbf{F}}$ .

### 6.4 Factor number estimation

Estimation of the factor number  $m$  can be achieved using information criteria. For stationary panels, the Bai-Ng criteria are of the general form

$$IC_p(k) = \ln \hat{\sigma}^2(k) + k \cdot g_{nT},$$

for penalty functions  $g_{nT}$  satisfying  $g_{nT} \rightarrow 0$  and  $\min(n, T) \cdot g_{nT} \rightarrow \infty$  under the asymptotic sequence selected. Here  $\hat{\sigma}^2(k)$  denotes the estimated variance of the estimated idiosyncratic component when  $k$  factors are included in estimation. Bai and Ng (2002) propose three penalties that satisfy these rate conditions under general large  $n$  and  $T$  asymptotics (i.e. both  $n \rightarrow \infty$  and  $T \rightarrow \infty$  without restriction on the relative rates of expansion). Specifically, they propose three penalty functions which we give in the Appendix.

The criteria are designed for stationary factors and idiosyncratic components. Bai (2004) derives different criteria for the case where the factors are  $I(1)$  but the idiosyncratic components are  $I(0)$ . Because there may be a mix of stationary and non-stationary factors in economic data, we apply the Bai-Ng criteria to first differenced data to ensure that the factors number is consistently estimated.

Model averaging can be used to reduce the effects of model uncertainty on inference and model prediction (Burnham and Anderson, 2002; Hjort and Klaesken, 2003). The

approach is commonly used in forecasting (Hansen, 2008) and inference (Hansen, 2007). In our empirical application the various penalty functions select very different factor numbers. Hence when estimating the trend component, we average across all model specifications using the Bai-Ng criteria as weights. See the appendix.

#### 6.4.1 Factor Model Selection Criteria

The specific Bai-Ng  $IC(k)$  model selection criteria are as follows.

$$\begin{aligned} IC_{p1}(k) &= \ln \hat{\sigma}^2(k) + k \cdot \frac{\ln(d_{nT})}{d_{nT}}, \\ IC_{p2}(k) &= \ln \hat{\sigma}^2(k) + k \cdot \frac{\ln(c_{nT})}{d_{nT}}, \\ IC_{p3}(k) &= \ln \hat{\sigma}^2(k) + k \cdot \frac{\ln(c_{nT})}{c_{nT}}, \end{aligned}$$

where  $c_{nT} = \min(n, T)$ ,  $d_{nT} = \frac{nT}{n+T}$ .

#### 6.4.2 Model Averaging

Let  $IC(j)$  denote the information criterion obtained with a  $j$ -dimensional model. The weight for model  $j$  is given by

$$w_j = \frac{\exp(-0.5 \times IC(j))}{\sum_{j=0}^{r_{\max}} \exp(-0.5 \times IC(j))}$$

Note that  $j = 0, \dots, r_{\max}$  corresponds to the number of factors, and  $r_{\max}$  is the maximum dimension of the model permitted.

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